

# PHYS 3B

# Lab Manual

First Edition (2022)

CoA Physics Department

## **NOTES FOR THE FIRST EDITION**

This first edition represents a collection of manuals for Physics 4B and 4C labs whose contents align with the topics covered in Physics 3B, electricity and magnetism, and modern physics. The format in which these manuals are included represents the form of the primary repository for these lab manuals, which are editable Canvas Pages maintained with the associated course shell. It is the intent of the physics faculty of CoA to, in near future, extensively adapt these labs and develop new labs of particular interest to biological science major students.

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# Lab: Intro to Physics Lab

*Note: This is a Newstyle Lab. Read each section of the lab manual carefully before starting to work on that section. Keep an organized record of any measurements you make, or any information you had to discover which was not apparent from the clear text of the lab manual. Make sure to stop at each boxed section of the lab manual and go through the activities in the boxed section. You will turn in a copy of your notes at the end of the lab section, and a follow-up lab narrative is due next week.*

Welcome to your first physics lab! In this introductory lab, we will introduce general structures and arrangements for physics lab and demonstrate some sources of error you may see in your future labs.

## Introductory Notes

If you have taken other science classes before, I hope you will soon see that physics labs are quite different. We say that "physics is the fundamental science," and what it means to me—your instructor—is that physics is the discipline where you learn to do a lot with very little. In other science classes, there are many facts, processes, and formulas to memorize; while there *are* facts, processes, and formulas to memorize in physics too, the emphasis of physics is not on those facts, processes, and formulas themselves but in understanding what the *fundamental laws* are—and how we might be able to derive those facts, processes, and formulas from the fundamental laws.

While I hope *you* will notice the differences soon yourself (and it has been many years since my last chemistry or biology lab, so you are in the better position to tell the differences), following are some key things I want you to note from the first day:

- **Physics lab is more open;** there is more room for experimentation, coming up with your own procedures, and trying out your own things. Of many reasons why physics lab *can* be more open, one important reason—especially important to keep in mind in your other labs—is that physics lab is safer. We rarely deal with chemicals or other dangerous substances; most of our equipment cannot cause grave bodily injury. While I do want you to take care with our instruments—please don't break them—because your risk of injury is low in general in physics labs, you can afford to be more flexible and more open in terms of lab procedures.
- **Physics lab is more precise and more accurate.** Even in this instructional lab setting with few sophisticated instruments, we can often achieve accuracy within 10% of the accepted numerical result—and with sufficient care and effort, 5% or 1% error is achievable on occasion. I want you to set a high bar for yourself. In physics labs, we deliberately choose situations with few complicating circumstances where this high precision can be achieved—and in the few situations where complicating circumstances remain, we want to better understand the complicating circumstances.

I hope you already understand the importance of experiment to sciences. You have already learned about the "scientific method" and the foundations of empiricism, describing how scientists—and everyone else—come up with hypotheses, design experiments to test the hypotheses, modify the hypotheses based on results of experiments, and iterate (that is, repeat these processes) these steps, improving the initial educated guesses.

At the core of physics problem-solving (or "puzzle-solving", in the language of Thomas Kuhn, a historian of science) is a trial-and-error approach to new situations and problems. "Trial" is the attempt at answering a question or situation that has come up; "error" is the *sophisticated* step in which you try to determine how close—or correct—your answer was. If you decide your answer wasn't quite right (or at least not as right as you would like it to be), then you modify your approach and do the "trial" again, hoping for an improvement at the "error" step. It is an iterative approach (the key feature of the scientific method) which values critical self-reflection.

Our highest goal this semester is to introduce you into physics problem-solving. Some of you might find that this *is* how you have been approaching problems in your life (I hope you will consider a career in physics!); others might find this approach challenging and often times frustrating ("Am I doing this right? Can you just tell me if this is right?"). Physics problem-solving requires *balancing* between logical deductions of mathematical reasoning (think back to when you were first taught about proofs in geometry, or possibly more recently in linear algebra, if you are farther along in your math sequences) and *intuitive* leaps—certain *key* steps in physics problem-solving are often poorly motivated (what this means is, if someone asks *why* I did something, I have no good answer, other than that doing that got me to the correct answer). This balancing is something you have to learn over time, and if by the end of the semester, you begin to understand what it is you are balancing, I will have achieved my goal.

This introduction into physics problem-solving is what we do in *both* physics lecture and lab, but it is far more important in lab. I'll be honest with you: in lecture, we don't really practice what we preach. You don't really get to apply the scientific method in lecture—you are not testing hypotheses when I teach you about Newton's Second Law ( $\vec{a} = \sum \vec{F}/m$ , for those who have seen it); a lot of time in lecture is spent teaching you about mathematical tools and problem-solving methods—which you *need to know*; you have to know what to "trial" in your trial-and-error approaches. It is in lab where you can *practice* being a scientist and an engineer. While we are not discovering new laws of physics in our physics lab (sorry, the equipment we are providing you with simply are not good enough), we are going through the same processes, reasoning steps, and practices that a practicing scientist or engineer would go through, as they tackle the problems at the frontier of our knowledge (using the state of the art equipment). It is through this lab that we hope you will grow in your thinking as a future scientist and engineer.

So, these are all high ideals—and I understand if these words don't yet have the same meaning to you as it did for me (well, when I was doing research as a grad student). We will start this journey together, and I will provide as many concrete instances as I can, where you can practice being a

scientist and an engineer, in anticipation of future careers you will take on, when you move on from here.

## Significant Figures, Uncertainty, and Percent Difference

Here is the first concrete example on practicing being a scientist. You have learned about significant figure rules in your earlier math and science classes. This is a version of these rules that you hopefully recognize:

(1) "significant figures" are the number of non-zero digits you have (plus some zero digits that you see that you can infer are significant); (2) when you add or subtract two numbers, you follow the significant figure of the number whose smallest significant figure is larger; (3) when you multiply or divide two numbers, you follow the significant figure of the number which has a smaller number of significant figures. Some examples are below:

- Following are some examples of significant-figure counting of numbers: 137 has 3 significant figures; 305 also has 3 significant figures; 305.0 has 4 significant figures; with 3050, it is ambiguous if it has 3 or 4 significant figures, but if you write it in scientific notation as  $3.050 \times 10^3$ , then it's clear that it has 4 significant figures. And 0.004 has only one significant figure (0.0040 has two significant figures). For examples below, we will stick to numbers that have some digits below decimal, so that there is not an ambiguity regarding number of significant figures.
- When you add 137.04 to 3.142, the answer, following the addition/subtraction rule is 140.18, so that the sum/difference has the same smallest significant figure as the larger smallest significant figure (137.04's smallest significant figure is 4, at hundredths; 3.142's smallest significant figure is 2, at thousandths; so the sum is rounded to hundredths).
- When you multiply 137.04 to 3.142, the answer following the multiplication/division rule is 430.6, so that the product/quotient has the same number of significant figures as the smaller number of significant figures (137.04 has five significant figures; 3.142 has four significant figures; so the product is rounded to have four significant figures).

So these are the rules you have learned and the rules you have been following. Now, as I will be telling you to no longer pay so close attention to these rules ("Down with the rules!"), it is important that you *understand* what these significant figure rules are trying to achieve.

**DISCUSSION** – Discuss with your partner or group to answer this question: What is the goal of significant figure rules? What do the rules (1) through (3) achieve?

Now, understanding what the rules were trying to achieve, here is why they are inadequate: the range of uncertainties covered by numbers that have the *same* significant figures is potentially large. Consider these two numbers, 1.1 and 9.9. They both have two significant figures. The first number 1.1 covers the range of numbers from 1.050 to 1.149 (they all get rounded to 1.1, when you round to

two significant figures). The second number 9.9 covers the range of numbers from 9.850 to 9.949. In many circumstances, the uncertainty you care about is *percent* uncertainty. Given the range of uncertainty (1.050 to 1.149, or 9.850 to 9.949, both covering a numeric distance of 0.099), what percent is that uncertainty of the overall number? It is in calculating this percent uncertainty that you will see that two numbers with the same significant figures can have very different percent uncertainty.

**QUESTION and ANSWER** – Calculate percent difference from 1.05 to 1.10 and calculate percent difference from 9.85 to 9.90. What range of percent uncertainties could you see in numbers with two significant figures? What range of percent uncertainties could you see in numbers with three significant figures?

In many contexts in this class, you will be told to use three significant figures. It's a "reasonable compromise." Most real numbers you will work with in this class have about 1% uncertainty, and using three significant figures ensure that you do not *increase* uncertainty in your numbers unnecessarily. Sometimes, when significant figure rules say a product should have two significant figures, you might see me retain three significant figures. Or sometimes, for intermediate calculation numbers, you might see extra number of significant figures being retained. This is so that existing *percent uncertainty* in the numbers are preserved and represented, and so that we do not increase the uncertainty unnecessarily.

Finally—for this introductory section—in laboratory context, you are going to see following three terms, sometimes being used interchangeably, sometimes being distinguished from each other: **percent uncertainty**, **percent difference**, and **percent error**. In most situations, they can be used interchangeably without resulting in serious error in meaning, but there are subtle differences in meaning. For example, a particular numerical result might have a zero percent error but *not* have a zero percent uncertainty. I hope you will see illustrative examples throughout this semester!

## Part A – Ball Measurements

This first part will provide you with a concrete example of importance of precision and an example of a situation where a given percent error in one measurement results in a different (greater, in this case) percent error in values that are calculated from that measurement. Following are the guiding Research Questions for this part. These are the questions you should answer—with all the necessary details and nuances—by the end of this part. Some description of the setup will follow below.

- **Research Question 1:** Do Nylon Balls (provided) have the same diameter as Stainless Steel Balls (provided)?
- **Research Question 2:** By what percent do volumes of Nylon Balls differ from volumes of Stainless Steel Balls?

We have two different physical objects available that are distinctively different in certain ways but are quite similar in some ways. In the classroom, you will find two containers, one of yellow nylon balls and another of polished stainless steel balls. One is plastic and the other is metal; the stainless steel ball is heavier than the nylon ball. There is one aspect in which they appear to be similar: size. I believe both of these balls are sold as "1-inch-diameter [MATERIAL] balls." What this means is these balls have a *nominal diameter* of 1 inch. So, does this mean these balls do have a diameter of 1 inch? There is a saying in physics: "[It] becomes a purely experimental matter."

We have several different measuring devices at your disposal. Using them (all, any, etc.), determine to your best ability the diameter of one nylon ball and one stainless steel ball. Pay attention to the significant figures ("1 inch", "1.0 inch", and "1.00 inch" carry different meanings; it might just be easier if you are measuring in centimeters or millimeters) and aim for the highest number of significant figures you can achieve given the measuring devices available. If you need help using some of the measuring devices (particularly for the caliper and the micrometer), please call me. The resource note at the end of this lab manual will link you to videos that demonstrate use of caliper and micrometer (in lab, call me; that's probably quicker than watching the video).

**MEASUREMENT** – Determine the diameters of a nylon ball and a stainless steel ball, to the highest precision possible given the available measuring devices.

*Note: There is an aspect of self-learning, or "inquiry-based learning" in these activities. I am deliberately not giving you a detailed set of steps to follow, so that you might figure it out for yourself. Having said that, my goal is not to frustrate you—if you feel stuck at any point, call me and ask me what the next thing to do is. I will figure out where you are and try to give you some ideas on what you could do next.*

The diameters you measure above have certain uses. Directly, they are used for quality control (most manufacturers of ball bearings specify not only the sizes of bearings but the tolerances of these sizes, so that their customers know how much—and how little—they can rely on the nominal values). More usefully in a physics lab setting, the value you measure will be the starting point of a calculation. Imagine you are trying to determine the density of a nylon ball. For the density, you need to know its mass and volume. And one of the ways to determine volume of a spherical object is by using this formula you may have seen in geometry:  $V_{\text{sphere}} = \frac{4}{3}\pi R^3$ , where  $R$  is the radius of the sphere. Having measured the diameter, you can calculate the radius easily and use that to calculate the volume of the ball, with better precision and certainty than just relying on the nominal values.

I'm hoping in your measurements above, once you got to high enough precision, you saw a slight difference between the steel ball diameter and the nylon ball diameter. In case your balls had the same *exact* diameter, down to tenths of millimeters, introduce a small error (change the last digit a little) for the purpose of below calculations.

**CALCULATION** – Calculate the percent difference in the volumes of the nylon ball and the stainless steel ball. Compare this to the percent difference in the diameters of the nylon ball and the stainless steel ball.

There is a way in which these percent differences are related (there is a calculus reason behind it, possibly to be covered in Calculus 2). If you want to know, please ask—knowing this particular reason isn't required for this class; I just want you to know the reason behind [error propagation](https://en.wikipedia.org/wiki/Propagation_of_uncertainty) ([https://en.wikipedia.org/wiki/Propagation\\_of\\_uncertainty](https://en.wikipedia.org/wiki/Propagation_of_uncertainty)).

The measurements you did in this section are the simplest type of measurements (length measurements) in a physics lab. The next section will involve precision measurements of *time*.

## Part B – Pendulum Period Measurements

Following are the research questions guiding the activities in this section. While it won't be until Chapter 15 that we can deal with the theory aspects of pendulum oscillations, everything you know about physics *now* is sufficient to perform measurements necessary to answer below research questions (again, with all the necessary details and nuances).

- **Research Question 1:** Does a pendulum swinging in 15-degree arcs have the same period as a pendulum swinging in 45-degree arcs?
- **Research Question 2:** What factors affect pendulum periods?

The pendulum clock, invented by Christiaan Huygens, is inspired by investigations of Galileo into pendulum motion. One (probably apocryphal) story tells of how a young Galileo became curious, looking at the motion of an incense-filled censer (not to be confused with sensors he had to contend with later in his life), that the back-and-forth swinging motion seemed to occur at regular intervals. Using his own heartbeat as a timekeeping device, he tried to confirm this observation.

This lab activity is inspired by the same physical setup (minus the incense, which could be irritating to those with allergies). You should find a simple pendulum setup, a small mass hanging at the end of a long string, at your table. The task at your hand is simple: measure the period of the pendulum. What is meant by "period" is the amount of time it takes for the pendulum to make one full back-and-forth swing (in general, "period" is the amount of time it takes for completion of shortest repeating motion). As an open investigation, this can be a little bit too open ("Do something with pendulums."), so I will ask you to focus on the following two questions. In order, first question:

**1. Does a pendulum swinging in a small range of motion have the same period as a pendulum swinging in a large range of motion?**

You can set a pendulum to move in a small range of motion by giving it a smaller initial angle of displacement from the vertical. The 15-degree arc described in Research Question 1 can be achieved by initially displacing the pendulum by 7.5 degrees from the vertical; when released, the

string of the pendulum will cover 15 degrees of motion. A "large range of motion" can be defined in a few different ways. A 16-degree arc is certainly larger, but such a small difference might make effect of a larger range difficult to measure. Research Question 1 uses 45-degree arc as a "large range" (to be achieved by initially displacing the pendulum by 22.5 degrees from the vertical), but if you thought you saw an effect of large range of motion on the period of the pendulum, nothing prevents you from trying larger amplitudes, up to about 180-degree arc or so (possibly smaller, if some mechanical connection/arrangement breaks with larger amplitudes).

You have timekeeping devices provided (and you are welcome to use your own, such as a stopwatch app on your smartphone). Devise your own experiments and take the necessary measurements.

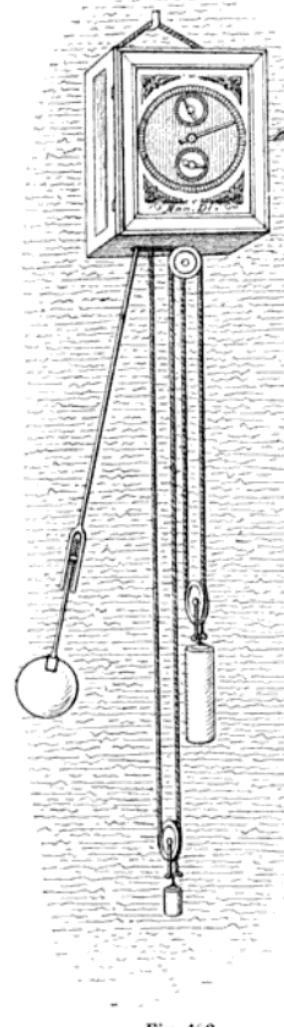
**MEASUREMENT** – Determine the effect of the size of the oscillations (small or large) on the period of the pendulum motion.

Having investigated the effect of the size of the oscillations on the period of pendulum motion sets you up for the second question:

## 2. What affects the period of a pendulum?

In particular, what changes can affect this period *strongly*? That is, imagine you wanted to change the period of the pendulum by a factor of 2 (either increase it by 2, causing it to oscillate slowly, or decrease it by 2, causing it to oscillate more quickly). What changes can you make to the pendulum to accomplish this? This is the place where I ask you to exercise your creativity—take a look at pendulum, taking stock of physical parameters that define this pendulum (what parameters are relevant? what parameters *aren't* relevant?). Imagine which of these parameters can be changed, change those parameters, and measure the effect of having changed those parameters.

There is an aspect of art and science to physics investigations. *Science* aspect here is the technical details involved in making the actual measurements. Hopefully having spent your time with Question 1 above, you were able to take a look at your measurement techniques and improve them (ask me if you want my critique!). The *art* here has to do with deciding what to investigate. Multidimensional parameter search is difficult (read more: [Curse of dimensionality](https://en.wikipedia.org/wiki/Curse_of_dimensionality) [↗](https://en.wikipedia.org/wiki/Curse_of_dimensionality) ([https://en.wikipedia.org/wiki/Curse\\_of\\_dimensionality](https://en.wikipedia.org/wiki/Curse_of_dimensionality)); here, "dimension" refers to one measurable aspect, not necessarily having anything to do with physical dimensions, such as three spatial dimensions, or different realms, such as "hell dimension"). There is art in deciding on the right thing to measure; this type of decision-making doesn't occur in a linear or deterministic fashion—so without spoiling the answer, I can't tell you what you should look for. (But if I need to spoil the answer in *individual cases*, I will, so feel free to ask!)



One thing I *can* tell you is how beneficial it is to work on the "science" aspect—if you have a good measurement technique; if you can quickly and efficiently determine, with the necessary degree of precision, what the period of the pendulum is, in a given amount of time, you can try more guesses. That will help you develop the "art" side of physics investigations.

**INVESTIGATE** – What factors affect the pendulum period significantly?

## Lab Narrative

That's it for the in-lab activities. Please turn in a copy of your notes (one *new* way I'm letting people turn in lab assignments: submit it online on Canvas as a high-quality photo of your notes; I can also help you make physical copies, if you don't have a good camera phone). This section will describe the **Lab Narrative** you should turn in next week.

## Purpose of the Lab Narrative

The purpose of the lab narrative is to *document your understanding of the lab*. All the details, information, and sections of the lab narrative should contribute to this goal. It might be easier to visualize this goal if you imagine a classmate (or a future Physics 4A student) reading your lab narrative and making sense of what you did and learned in the lab. Feel free to omit superfluous details that you feel does not contribute to documenting how you understood the lab.

When someone—particularly me, your instructor and grader—reads your lab narrative, that someone should feel that you comprehended elements of the lab that you could comprehend within the given time and equipment limitations.

## Organization of the Lab Narrative

Following is how I recommend that you organize your lab narrative. You *might* find it necessary to organize it differently; feel free to experiment. The structure given here is the basic structure in which most physics research papers are published in (activity we are trying to model after); it's not a strict guideline you must follow.

- **Introduction:** The introduction section usually gives a background to the experiment, explaining why you are doing the experiment. This might borrow moderately from the lab manual. Introduction is *usually* the last section that should be written (because what you learn from completing the other sections might inform how you write the introduction).
- **Methods and Procedure:** There should be a section in which technical details are described. If someone else wanted to replicate your experiment, what would they need to know? Did you find that you had to wrestle with any particular problem? The procedural details here should give enough details so that others could follow them in order to reproduce your results, but it should skip the boring bits.



- **Results and Analysis / Discussion:** Sometimes information in this section can go alongside Methods and Procedure. Other times, especially if there is a substantial amount of analysis needing to be done, it might be organized into its own section. It's your call, really—does it seem most natural to cite the results of your measurements and what these measurement mean alongside the description of experimental methods, or should the discussion of results be separated out into its own section? Experiments calling for extensive error analysis might need its own results and discussion section, to discuss all possible sources of error.
- **Conclusion:** Conclusion should be written in a way so that if that is the only section someone read, they can quickly get a sense of what you found out in your experiment. It might be a summary of results section; it might boldly state some new (or at least surprising) thing you discovered in your experiment. This is also a good section in which to propose future work—what would you have done, if you had additional time and wanted to further investigate what you were investigating?

## Length of the Lab Narrative

There is no particular length requirement on the lab narrative. But just to set proper expectations, for most labs in this course, a high-quality lab narrative including all relevant details and thoughtful analysis can be written in about 3 pages, single-spaced, inclusive of any figures or table of data.

If you feel it's necessary, you may write longer lab narratives, but it is not necessary to write longer lab narratives just to fill space.

## Resources and Additional Information

This section is an appendix of information, concepts, techniques, and information that you might find useful for this (and future) lab. I will happily answer any questions *in lab*, but especially in case you feel you need information provided here as you are doing your work outside the lab (either preparing for lab or writing your lab narrative), this is provided here.

### A: Percent Uncertainty, Percent Difference, and Percent Error

The terms **percent uncertainty**, **percent difference**, and **percent error** comes up often, and I thought it would be useful to have a single place in which to describe them in some detail, *particularly* moving beyond the simplistic notions of percent error. The simplistic notion of percent error is what you might have seen in previous science labs: "Percent error is the difference between the theoretical value and experimental value, divided by the theoretical value." This simple description is not wrong (just as the significant figure rules you have been following weren't wrong); it just doesn't cover all the cases you might see. Now is the time you learned *why* we calculate percent error and learn different, flexible approaches to expressing experimental uncertainties.

*Every physical measurement has an uncertainty associated with it.* Sometimes these uncertainties are explicitly stated and easy to infer; other times it takes fair amount of work determining the

uncertainty associated with each measurement. Imagine making a measurement of  $g$ , which describes gravitational field near the surface of the Earth. The commonly accepted value of  $g$  is  $g = 9.81 \text{ m/s}^2$ . There are different ways of measuring  $g$ . One way is to observe motion of an object in free fall and calculate its acceleration. If I did this measurement and obtained a value of  $9.8 \text{ m/s}^2$ , I can ask and answer three different questions: (1) what is the percent uncertainty in this measurement, (2) what is the percent difference in this measurement, and (3) what is the percent error in this measurement?

The percent difference (2) is the easiest value to calculate with minimal discussion, so I will do that first. With percent difference, often you have two values in mind. In this case, it would be the commonly accepted value  $9.81 \text{ m/s}^2$  and the experimentally measured value  $9.8 \text{ m/s}^2$ . So I take the difference, obtaining  $0.01 \text{ m/s}^2$  (note that I am ignoring a significant figure rule to do this; I made the judgment call that this is preferable to reporting a difference of  $0.0 \text{ m/s}^2$ , following the significant figure rule blindly). To obtain a percent figure, I have to divide this by a reference number, and here, I will choose my experimentally measured value, which gives me  $0.01/9.8 = 0.00102 = 0.102\%$ . There are two other numbers I could have chosen as a reference number, either the commonly accepted value, or an *average* of all available numbers. Usually when the percent difference is small, this choice of reference number does not significantly affect your final result, so beyond being explicit about how I am calculating the percent difference, I don't worry so much over which reference number I have chosen.

The percent *error* (3) is more subtle, because you are making a judgment call on which number is the correct, "true" value. If described as percent error, rather than blindly using the commonly accepted value of  $9.81 \text{ m/s}^2$ , I would need to investigate to see what value I *should have* measured.

The *theoretical* value of  $g$  varies depending on your latitude (from  $9.832 \text{ m/s}^2$  at the poles to  $9.780 \text{ m/s}^2$  along the equator). And there maybe local differences (such as mineral deposits) which affects the "true" value of  $g$  you "should have" measured.

I hope my biases are showing with these scare quotes—especially in our context, it's much easier to deal with percent differences. With percent differences, you can simply report difference between two numbers (one you measured, and another that you are using to see if your number is reasonable) without needing to answer the difficult question of what exact number you should have measured. (Having said that, I won't be too strict about enforcing *correct* usage of the term "percent error.")

The percent uncertainty (1) is the one that needs the greatest consideration, and the frank truth here is, I don't have enough information to give percent uncertainty of the value measured,  $9.8 \text{ m/s}^2$ . I would need to know the details of experimental methods to have an educated guess at the percent uncertainty. Based on significant figure rules, I do hope the percent uncertainty is not much higher than  $0.1 \text{ m/s}^2$ , because the "8" is supposed to be a significant figure, and if the true value was lower than  $9.7 \text{ m/s}^2$  or higher than  $9.9 \text{ m/s}^2$ , then that "8" is not very significant. The percent uncertainty is not simply a measure of how far your number is from the accepted value (that's "percent difference"). The percent uncertainty expresses how confident ("certain") you are of your measurement. For an

example, when I measure length of anything I can put ruler against with a ruler, my uncertainty in that measurement is about 0.5 mm (1 mm being the smallest visible mark on most rulers). If I can't place the ruler right against the object, depending on the setup, my uncertainty might go up a little, maybe as high as 3 mm (depending on my considered judgment on how well I could judge the location of ruler markers with my eyeballs). It doesn't mean I am wrong by 3 mm (or 0.5 mm when I have ruler right up against the object); if I knew how much I was wrong by, I would correct for it. I am simply certain that whatever the correct value is, it is within the 3 mm of the value I am reporting—in other words, my level of uncertainty is 3 mm for that particular measurement where I could not place a ruler against the object.

You can see expressions of this type of uncertainty when you look up physical constants. For example, when I look up  $G$ , this is the value I see:  $G = 6.67430(15) \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ . What this means is, those who compiled this result are confident that the first four digits, 6.674 are correct. The last two digits, 3 and 0 represent the best estimate, and the number in the parentheses represents the level of uncertainty. According to those who compiled this result, the value of  $G$  may be as small as  $6.67415 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ , or as large as  $6.67445 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$  (within some confidence intervals, for those of you who took statistics).

This is the considered, *correct* notion of uncertainty, and "percent" part simply turns this figure into percentage terms, by dividing the uncertainty by a reference number (usually your best estimate). So in the example of  $G$  above, the percent uncertainty in the accepted value of  $G$  is  $0.00015/6.67430 = 2.25 \times 10^{-5} = 0.00225\%$ .

In our labs, we often use the percent difference as a way to guess at the percent uncertainty. This is a mere shortcut (we have limited time in lab); study of experimental uncertainty requires more consideration.

## B: Measurements with a Caliper/Micrometer

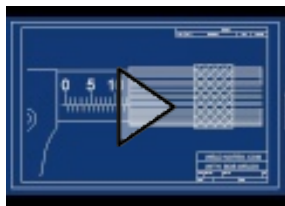
In lieu of written descriptions in use of a caliper or a micrometer, let me refer you to these videos. I can also demonstrate these techniques in lab (and this is where the information is not very useful outside of a lab setting, unless it's to tell you that you did it wrong in lab).

[How to read a metric Vernier caliper](https://youtu.be/vkPlzmalvN4)  [. \(https://youtu.be/vkPlzmalvN4\)](https://youtu.be/vkPlzmalvN4)



[. \(https://youtu.be/vkPlzmalvN4\)](https://youtu.be/vkPlzmalvN4)

[How to read a micrometer](https://youtu.be/StBc56ZifMs)  [. \(https://youtu.be/StBc56ZifMs\)](https://youtu.be/StBc56ZifMs)



(<https://youtu.be/StBc56ZifMs>)

## C: Measuring Your Reaction Time

The human reaction time is a common source of error whenever manual measurement of time interval is involved. This is one particular source of error you need to be mindful of, as you are performing the pendulum measurements. Below procedure describes how you can measure your own reaction time, with the equipment available in lab. If you just want a quick number you can use, the typical human reaction time involving visual information (you see a meter stick falling) and hand movement (you close your fingers) is about 0.2 second.

**Procedure** (an example data table you can use with this procedure is provided below):

- Have your lab partner hold a meter stick between your thumb and forefinger.
- The thumb and forefinger should be about 2 cm apart (about twice the thickness of the meter stick) so that they do not touch the meter stick.
- Record the location of the thumb and forefinger before dropping the meter stick (Starting Point, in the table).
- Your lab partner will drop the meter stick without warning. Try to catch the meter stick as quickly as possible. If you catch it before the meter stick is dropped or if your arm moves as you catch the meter stick, repeat the measurement. You can brace your arm using the table to make sure that your arm will not move down as you catch the meter stick.
- For each successful measurement, record the end finger position (Ending Point, in the table).
- Calculate the distance traveled, and convert it to meters. Record it on the table as Distance Traveled.
- The following is a kinematics formula we will cover in class. For an object in freefall, starting from rest, the distance traveled is given by,  $d = (1/2)gt^2$ , where  $g = 9.8 \text{ m/s}^2$ . Solving this expression for time  $t$  gives,  $t = \sqrt{2d/g}$ , and plugging in the numbers ( $d$  being distance traveled in meters) gives the reaction time,  $t$ . Use this to fill in the last column.

Below table provides enough space for 5 trials. By averaging the 5 trials, you can obtain a better estimate of your reaction time, *and* the spread of the points will give you a way to estimate an uncertainty of your reaction time (think through it).

| Trial | Starting Point (cm) | Ending Point (cm) | Distance Traveled (m) | Reaction Time (s) |
|-------|---------------------|-------------------|-----------------------|-------------------|
| 1     |                     |                   |                       |                   |
| 2     |                     |                   |                       |                   |
| 3     |                     |                   |                       |                   |
| 4     |                     |                   |                       |                   |
| 5     |                     |                   |                       |                   |

*Note: This is a Newstyle Lab. Most other labs you will see in the semester are Oldstyle Labs, which will appear quite a bit more structured. Please do follow instructions in those other labs; what remains the same with Newstyle Labs and Oldstyle Labs are my expectations—remain flexible and be open to creative problem-solving in physics labs; if you aren't sure of something, always ask!*



# Worksheet Lab: Electric Fields and Flux Manual

*Note: For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on separate pieces of paper to turn in. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.*

## Introduction

The goal of this worksheet lab is to help you gain some physical intuition for electric fields and flux, leading up to an understanding of Gauss's Law at an intuitive level. While it is easy to memorize a few formulas for electric fields produced by different geometries of charge distribution, your chance at successfully understanding theory of electromagnetism will be greatly enhanced by understanding the concepts of fields and their sources at a fundamental level.

This worksheet lab is in a few parts, starting with introduction to electric fields (picking up where we left last week), leading you through discussion of field lines, electric flux, and ending with introduction of Gauss's Law.


## Introduction to Electric Fields

Electric fields ( $\vec{E}$ ) are defined by the equation,

$$\vec{F}_e = q\vec{E} \quad (\text{Eq. 1})$$

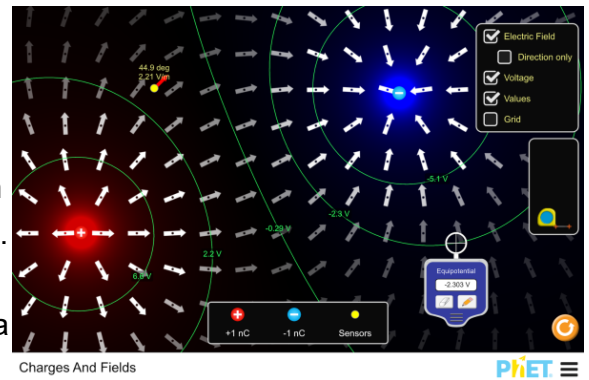
where  $\vec{E}$  is the net electric field at the location of charge  $q$ , and  $\vec{F}_e$  is the net electric force on charge  $q$ . At least for our current purposes, it is nothing more than a mathematical device to separate the calculation of  $\vec{F}_e$  into two parts: (1) figuring out the net effect of all other charges (calculating  $\vec{E}$ , independently of  $q$ ), and (2) having this field act on the charge  $q$  to find the net electric force. In this context, we often call the charge  $q$  a "test charge", because it does not factor into the calculation of  $\vec{E}$ , and its purpose is limited to experimentally "testing" what the value of electric field is at the location it's at, by measurement of electric forces on the test charge.

So, for the rest of this worksheet lab, we are not going to refer to electric forces (with *few* exceptions). But when asked to figure out directions of electric field at a location, imagine what would happen to a *positive* test charge placed at the location. Whichever direction a positive test charge will get pushed/pulled is the direction that electric field will point in.

It helps to have a correct mental image for the electric field. Use the PhET simulation, [Charges and Fields](https://phet.colorado.edu/en/simulation/charges-and-fields)  (<https://phet.colorado.edu/en/simulation/charges-and-fields>), to help you start building this image.

Start by following the set up instruction below, in order to avoid an information overload (and skipping thinking about electric fields concretely):

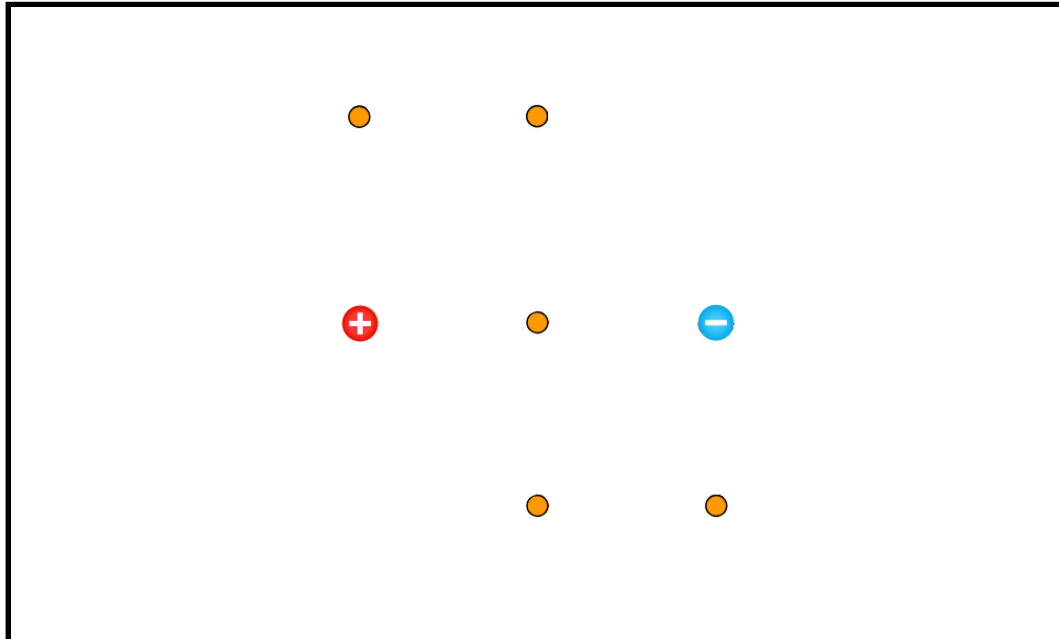
1. Launch the simulation. You should see a black blank screen with some options and repositories for charges and sensors.
2. Uncheck "Electric Field".
3. Place a couple sensors on the screen. You should just see a yellow dot with nothing much around it.
4. Place a single **+1 nC** charge on the screen somewhere. You should see red arrows sticking from the yellow dot, indicating the strength (length of arrow) and direction (direction of arrow) of electric field at the location of the sensor.



**Q1:** Move the sensors around the **+1 nC** charge. How would you describe the electric field (direction and magnitude) in relation to the **+1 nC** charge? Write down a succinct description in your lab report.

**Q2:** Remove the **+1 nC** charge and place a **-1 nC** charge. Also move the sensors around the **-1 nC** charge. How would you describe the electric field (direction and magnitude) in relation to the **-1 nC** charge? Write down a succinct description in your lab report.

Let's make it interesting. Remove all sensors from the screen, and place two charges, one **+1 nC** and another **-1 nC** near each other, as shown in the screenshot below. The small dots are sensors for Q3 and Q4 (don't place them yet).




**Q3: Prediction** Before you place any sensors (and with "Electric Field" box still unselected), sketch electric fields at each of the locations of the sensors above (you may use this page of lab manual, or reproduce the drawing above in your lab report). Pay especially close attention to the directions and magnitudes of the electric field.

**Q4: Test** Place the sensors as shown above. Compare the electric fields indicated by sensors with the electric field vectors you have drawn. Note and explain any discrepancies.



When you have discussed your results in Q3 and Q4 within your group, call me to get my initial on your lab report. **Lab reports missing instructor's initial will lose points.**

## Field Lines

Now check "Electric Field" box to show the electric fields that we have been hiding (if you want, before you move on, check out the views with Q1 through Q4 above, with "Electric Field" turned on, to check your answers in Q1 through Q4). The PhET simulation shows a [vector field](https://en.wikipedia.org/wiki/Vector_field)  ([https://en.wikipedia.org/wiki/Vector\\_field](https://en.wikipedia.org/wiki/Vector_field)) view of electric fields. This is easy to visualize in computer software (the space is divided up into lattice points, the electric field vector is calculated at each lattice point, and vector is illustrated at each point, using some suitable convention to indicate the magnitude) but difficult to draw by hand. Also, illustrating electric fields as an unconstrained vector field misses some important details about properties of electric fields.

The electric field lines are the graphical aid to overcome both of these issues: (1) they can be drawn relatively easily by hand, and (2) carefully drawn electric field lines will conform to known rules of electricity. Intuitively, electric field lines can be drawn from the vector field representation by starting a line near the neighborhood of a positive charge, drawing the line without lifting the pen following the direction of vector field at each location. When drawn carefully with some sensible rules (for example, drawing a reasonable number of lines, like 4 or 8 for each positive charge), the electric field lines follow below rules from OpenStax *University Physics*:

### Problem-Solving Strategy: Drawing Electric Field Lines

1. Electric field lines either originate on positive charges or come in from infinity, and either terminate on negative charges or extend out to infinity.
2. The number of field lines originating or terminating at a charge is proportional to the magnitude of that charge. A charge of  $2q$  will have twice as many lines as a charge of  $q$ .
3. At every point in space, the field vector at that point is tangent to the field line at that same point.
4. The field line density at any point in space is proportional to (and therefore is representative of) the magnitude of the field at that point in space.
5. Field lines can never cross. Since a field line represents the direction of the field at a given point, if two field lines crossed at some point, that would imply that the electric field was pointing in two different directions at a single point. This in turn would suggest that the (net) force on a test charge placed at that point would point in two different directions. Since this is obviously impossible, it follows that field lines must never cross.

**Q5:** Draw a simple set of electric field lines for a positive charge (use the PhET simulation to create a vector field representation which will serve as a guide). Verify that the electric field lines you drew follow the rules above (#1, 3, 4, and 5; there isn't a way to verify #2 with a single charge).

**Q6:** Draw electric field lines for the charge arrangement in Q3 and Q4. Again, verify that the electric field lines follow the rules above (#1, 2, 3, 4, and 5).

**Q7:** Let's make it interesting. Increase the amount of positive charge from Q6, so that you have **+2 nC** of charge on the left. Draw electric field lines (where necessary, refer to vector field illustration from PhET simulation). Make sure your electric field lines follow the rules #1, 2, 3, 4, and 5 above.

Electric field lines can also be used to represent electric fields from extended charge distributions, such as an infinitely long line of charges (of linear charge density  $\lambda$ ) or an infinite plane of charges (of surface charge density  $\sigma$ ). The electric fields for these geometries are given by (you will derive this later this week using Gauss's Law):

$$\begin{aligned} E_{\text{line}} &= \frac{\lambda}{2\pi\epsilon_0 r} = 2k_e \frac{\lambda}{r} \\ E_{\text{plane}} &= \frac{\sigma}{2\epsilon_0} = 2\pi k_e \sigma \end{aligned} \quad (\text{Eq. 2})$$

Here  $r$  stands for the distance from the line; the direction of electric field points away from the charges (radially outward from the line, or perpendicular to the plane).

**Q8:** Sketch electric field lines for (a) an infinitely long line of charges and (b) an infinite plane of charges. Draw multiple views to correctly and completely represent the *three-dimensional* electric field lines. Check your sketches against the rules above (#1, 2, 3, 4, and 5), and discuss your drawings with your group.

When you have discussed your results in Q8 within your group, call me to get my initial on your lab report. **Lab reports missing instructor's initial will lose points.**

Note that there *are* some situations involving continuous charge distributions (most notably a uniform sphere) where it is not easy to draw electric field lines. It is generally useful (and possible) to draw electric field lines for *regions without electric charges* (or "vacuum").

## Electric Flux

Above exercises are really building up to this point: how do you calculate electric flux? Electric flux is defined rather simply,

$$\Phi_E = \vec{E} \cdot \vec{A} \quad (\text{Eq. 3a})$$

or if the electric field varies over the area, then as an integral over the surface,

$$\Phi_E = \int \vec{E} \cdot d\vec{A}. \quad (\text{Eq. 3b})$$

The dot product is the same dot product you have seen in Physics 4A; area is made into vector by associating it with the direction of the normal vector:

$$\Phi_E = \int \vec{E} \cdot \hat{n} dA. \quad (\text{Eq. 3c})$$

Now, here's a little secret: in this class, you are going to *very seldom* calculate electric flux using the above formula. Very often (in application of Gauss's Law), we will find some way not to actually do the integral. This is why it is so important that you understand flux intuitively—it is easy to simply chug through formulas if you are given an explicit integral to do; it is harder to intuitively figure out the answer when you have no explicit formulas to actually work through.

So, please try below exercises to help you develop your intuition for electric flux.

**Q9:** Let's start with an easy case. Suppose that a square of area  $A$  is placed in a region of uniform electric field  $E_0$ , oriented so that it is perpendicular to the electric field. In your lab report, sketch this picture and calculate the electric flux for this situation.

**Q10:** Let's modify the case in Q9 a little bit by rotating the square. Defining our coordinate axes so that the electric field points in the direction of  $x$  (so that  $y$  and  $z$  axes are perpendicular to the electric field), imagine rotating the square by  $20^\circ$  with the  $y$  axis as the axis of rotation. Sketch this picture and calculate the electric flux for this situation.

**Q11:** Without taking the square out of the region of uniform electric field, would it be possible to arrange it so that the electric flux through the square is zero? Describe such an arrangement.

With that practice out of the way, let's consider more realistic (and relevant) situations.

**Q12:** Consider a positive point charge of magnitude  $q$ . What is the net flux out of a sphere of radius  $R$ , with the charge  $q$  at the center? Choose  $\hat{n}$  so that on every point on the sphere,  $\hat{n}$  points outward (this will be our normal convention for *closed* surfaces the rest of this semester). Show your work with the explicit calculation. Make sure to cancel out anything that cancels out (and show the cancellation).

Above calculation should have been doable and show a surprisingly simple result at the end. Now, please consider and answer below two follow-up questions (no detailed calculation, especially those involving integrals, is needed).

**Q13:** Consider the following change to the setup in Q12. If the sphere has been placed so that the charge is not at the center but  $R/2$  away from the center, could you do a calculation similar to Q12? Why not? (Bonus Point: And given that you cannot do a similar calculation, can you nonetheless *intuitively* guess what the answer would be? If you are not sure, try drawing an electric field line diagram.)

**Q14:** Consider the following change to the setup in Q12. If an additional charge  $-q$  has been placed at a distance of  $2R$  from the charge  $q$  (so it is outside the sphere but not very far away), could you do a calculation similar to Q12? Why not? (Bonus Point: And given that you cannot do a similar calculation, can you nonetheless *intuitively* guess what the answer would be? If you are not sure, try drawing an electric field line diagram.)

## Epilogue: Gauss's Law

The questions in Q13 and Q14 are my attempts at leading you to Gauss's Law on your own. Gauss's Law, which says,

$$\oint \vec{E} \cdot d\vec{A} = Q_{\text{encl}}/\epsilon_0 = 4\pi k_e Q_{\text{encl}}, \quad (\text{Eq. 4})$$

is actually quite *intuitively* true, if you understand it as saying this: "electric charges are sources of electric field lines" (field lines which obey the rules we gave above). If you enclose the "sources" within a closed surface, all you need to know to figure out how much "flows out" (net outward electric flux) is knowing how much "flows in" through the sources, provided that the thing that "flows" is conserved. (There are "scare quotes" around all the "fluid" language, because electric field doesn't actually flow; it's just that the imagery—which is quite natural, considering how the electric field lines look—meshes really well with the language of fluids and flow.)

Now, we are going to spend much of remainder of this week learning about how to use Gauss's Law to find electric *field* (not flux; we seldom actually care about electric flux; it's the electric field that relates directly to electric force). So, with that in mind, let me end with this question.

**Q15:** With Gauss's Law, it's quite clear that you *can* calculate electric flux in scenarios given in Q13 and Q14. This is the question: can you, from knowing the electric flux through the spherical surface, find the magnitudes and/or directions of electric field on the spherical surface in either of the scenarios in Q13 and Q14? If yes, describe how; if no, explain why not.

# Lab: Van de Graaff Generator Manual

*Note: For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on separate pieces of paper to turn in. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.*

## Introduction

With the introduction of voltage (electric potential), we have introduced all the concepts (and language) we need to describe static electricity. But although you should be already familiar with some of the terms discussed (such as electric force), many of the concepts introduced so far may seem abstract and difficult to relate to (electric field and voltage, for example).

The purpose of this lab is to give you some hands-on experience dealing with static electricity, and using the language developed so far to describe qualitatively and quantitatively what happens as an object acquires electric charge. The lab is in three parts: (a) simple experiments using pith balls, (b) calculation/estimate with Van de Graaff generator, and (c) simulation exercise for gaining familiarity with equipotentials.

This is not a precise lab, and no error analysis is required (but you will estimate some numbers within an order of magnitude). Please focus on conceptual understanding of topics so far covered in lecture.

## Part A: Experiments with Pith Balls

You have a couple pith balls on strings. Some pith balls are provided with a stand. If you do not have a stand for your pith ball, tape the midpoint of strings to a rod for the exercises below. Follow directions below and give your answers in the provided space.

Using the provided material (glass or plastic rod, and a set of materials to rub them with: silk, plastic bag, or fur), give a static electric charge to the rod and observe its interaction with the pith ball. Answer Q1 below.

*Note: Because of small amount of charges involved, whether exercises here can be done successfully depends on a couple factors, including the weather conditions and if your rubbing material already has too much excess charge. Call me if you are having difficulty seeing an interaction between the rod and the pith ball.*

**Q1:** Observe and explain interactions between charged rods and pith balls. What happens when a charged rod is brought near an uncharged pith ball? What happens if a pith ball is left in contact with a charged rod long enough? [If you see nothing interesting happening, charge up the rod again and

bring it to contact with pith ball until you see something; if you continue to see nothing, call me.] Explain in your lab report what you see in terms of charges (and movement of charges) in and around the pith balls.

Now, using the Van de Graaff (VDG) generator, you are going to charge up the pith balls and observe the interaction between two charged pith balls. Answer the questions below (and please share the VDG generators, as there are only two for the class).

**Q2:** Derive an expression needed for this portion of the lab. You are going to estimate the amount of charge on the pith balls by measuring the maximum separation between the balls. Assuming that the two pith balls have the same amount of charge on them, derive an expression (using the Standard Strategy!) that relates the charge  $Q$  on each pith ball to the separation distance  $d$  between the pith balls, as they hang from a common point, each with a string of length  $L$ . Measure any parameter values you need for this expression; look up any physical constants needed. Show your work in your lab report, and draw figures as needed. (Call me if you are not sure about the correctness of the formula you derived.)

**Q3:** Using the VDG generator, charge the pith balls (allow the pith balls to touch the VDG generator for as long as possible—be careful, as they will bounce around). Quickly measure the separation distance  $d$  between the pith balls. Try a few times. As the pith balls do not remain charged for a long time, use the maximum distance you measure. Record the distance  $d$  in your lab report (record the value you will use for your calculation).

**Q4:** Using your formulas and measurement above, answer these questions: (i) What is the maximum charge (in coulombs) that a pith ball holds? (ii) What is the maximum number of excess/lack of electrons that a pith ball held? Is this a significant number of electrons, compared to the number of atoms in a pith ball? (Make best estimates, using a periodic table and a table of constants.) (iii) How much mass does the excess/lack of electrons account for? Is this a significant amount of mass?

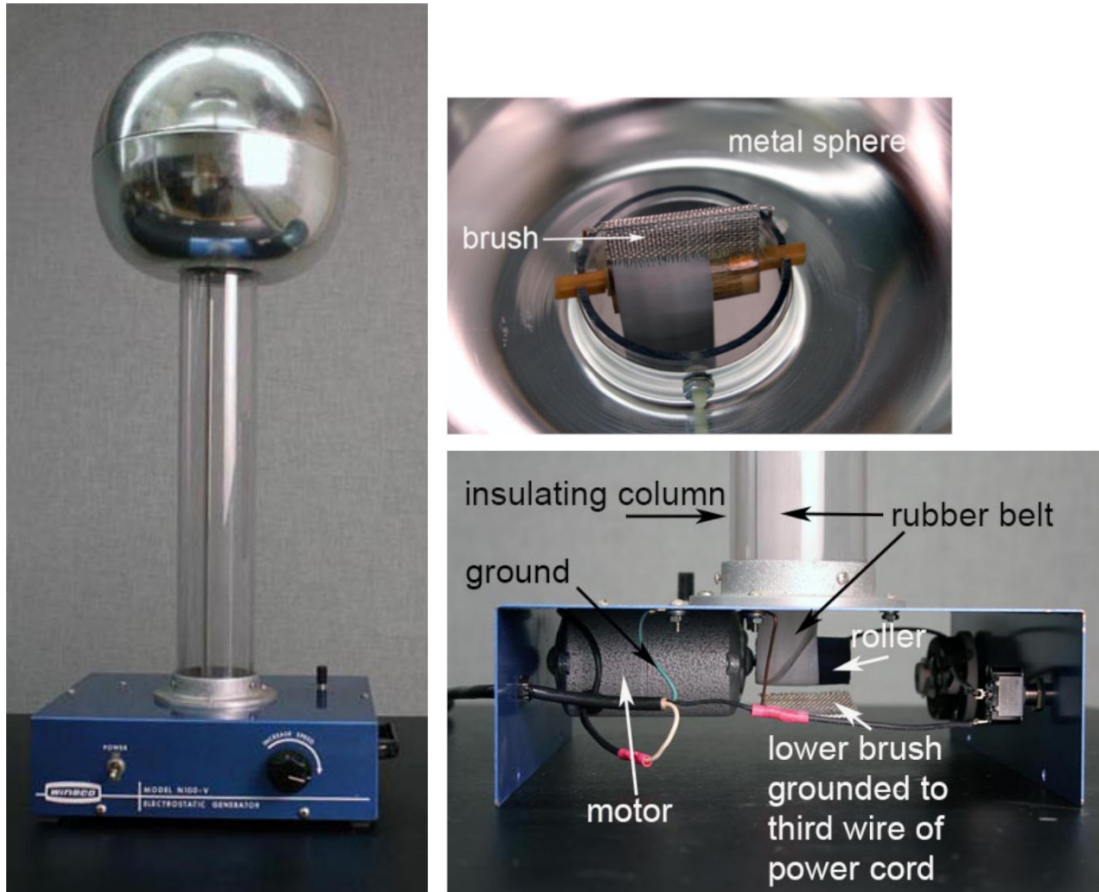
## Part B: VDG Charge Accumulation Rate

VDG generator is used in introductory physics classroom as a safe high-voltage source. The estimation exercises below will show why VDG generator is safe to use, even though it provides many thousands of volts.

**Some details of VDG generator** (Photos from

<http://demoweb.physics.ucla.edu/content/experiment-4-van-de-graaff> ↗

(<http://demoweb.physics.ucla.edu/content/experiment-4-van-de-graaff>)



Make sure your VDG generator setup is set up as described below.

- Set up a large conducting sphere next to your VDG generator. Ground the wire connected to the conducting sphere by plugging it into a ground connection provided on your VDG generator. (Ask me if you are not sure.)
- Set up your VDG generator and conducting sphere close to each other and turn on the VDG generator. You should hear (and maybe see) a spark between the two spheres.
- Slowly move away the conducting sphere. You will hear and see less frequent, larger sparks. Set up the conducting sphere at a maximum distance where you see a spark between them (if you set them up too far apart, VDG generator will discharge elsewhere; stay as far from VDG generator as possible).

**Q5:** Measure and record the following. Show your measurement and work in your lab report: (i) As the VDG generator is running at maximum speed in the setup described above, measure the rate of sparking (# of sparks per second). (ii) After turning off VDG generator and grounding the conducting sphere and the VDG generator (so that you don't shock yourself), measure the "distance" between VDG generator and the conducting sphere. For (ii), you need to decide which distance it is most relevant for you to measure; you will be interested in the voltage difference between the conducting sphere and the VDG generator.

**Q6:** Using your knowledge of static electricity, estimate how much charge accumulated on VDG generator before sparking happened between VDG generator and the conducting sphere. Give your estimate in coulombs, and show your work in the space below. Please consider the following as you estimate the maximum charge on VDG generator: (i) The dielectric breakdown of air happens at about electric field strength of 3,000,000 V/m. (ii) From knowing maximum possible electric field, how can you relate that to maximum possible voltage difference? (iii) Do you know the voltage of conducting sphere? Of VDG generator? (iv) Knowing electric field and/or voltage, how do you relate the amount of charge (approximated as uniform spherical shell) to the electric field and/or voltage?

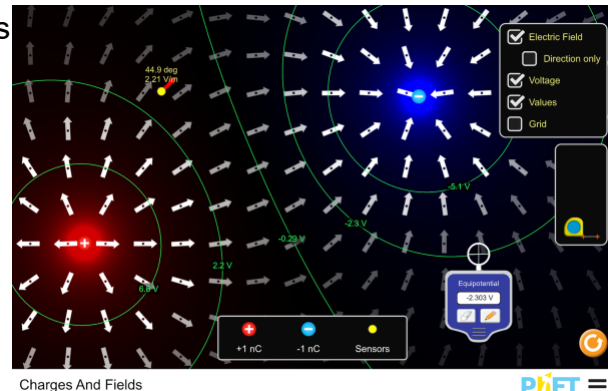
**Q7:** Using your measurement in Q5 and derivation in Q6, estimate the rate of charge accumulation on VDG generator in coulombs per second. Consider and answer these questions: (i) A current (rate of charge flow) of about 0.1 coulombs/sec can be large enough to disrupt a person's heartbeat and kill him (<http://www.darwinawards.com/darwin/darwin1999-50.html>). Does VDG generator generate enough current to kill a student (or instructor)? (ii) What approximations or assumptions did you have to make in your calculation? How far off do you think your estimate could be (and if you can tell, in what direction)? Does this consideration of estimate uncertainty change your answer regarding lethality of VDG generator?



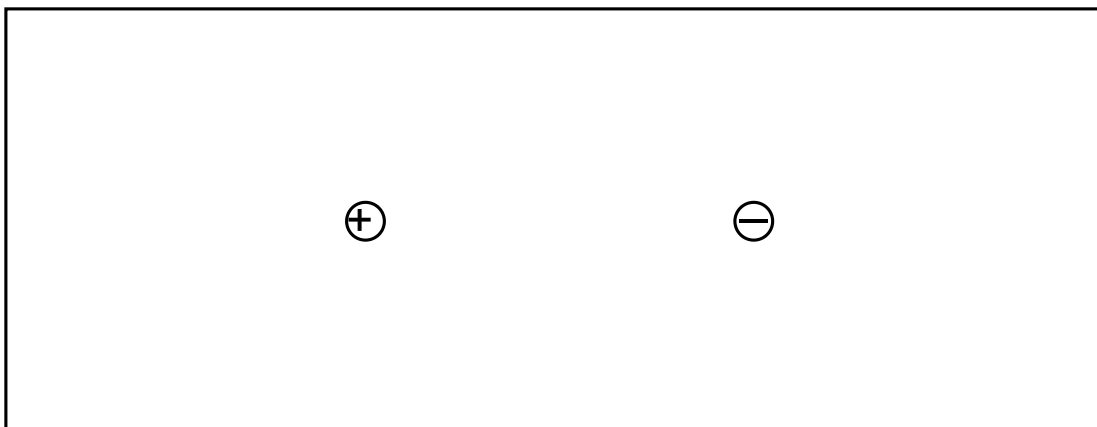
## Part C: Equipotentials

Use the remaining time in class to answer below questions relating to equipotentials. You are encouraged to use the Charges and Fields simulation on PhET website:

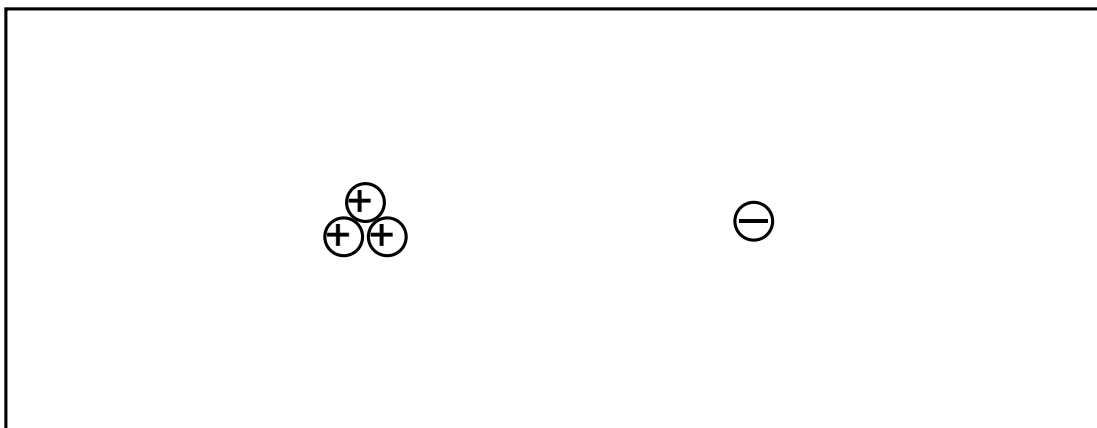
<http://phet.colorado.edu/en/simulation/charges-and-fields> (http://phet.colorado.edu/en/simulation/charges-and-fields)



**Q8:** For a warm-up: Sketch electric field lines (using at least 8 field lines per charge) and equipotential lines (using at least 8 equal divisions of voltage) for a dipole charge distribution shown below. If it helps, use the PhET simulation to see what the field lines and equipotentials look like and copy over the sketch.



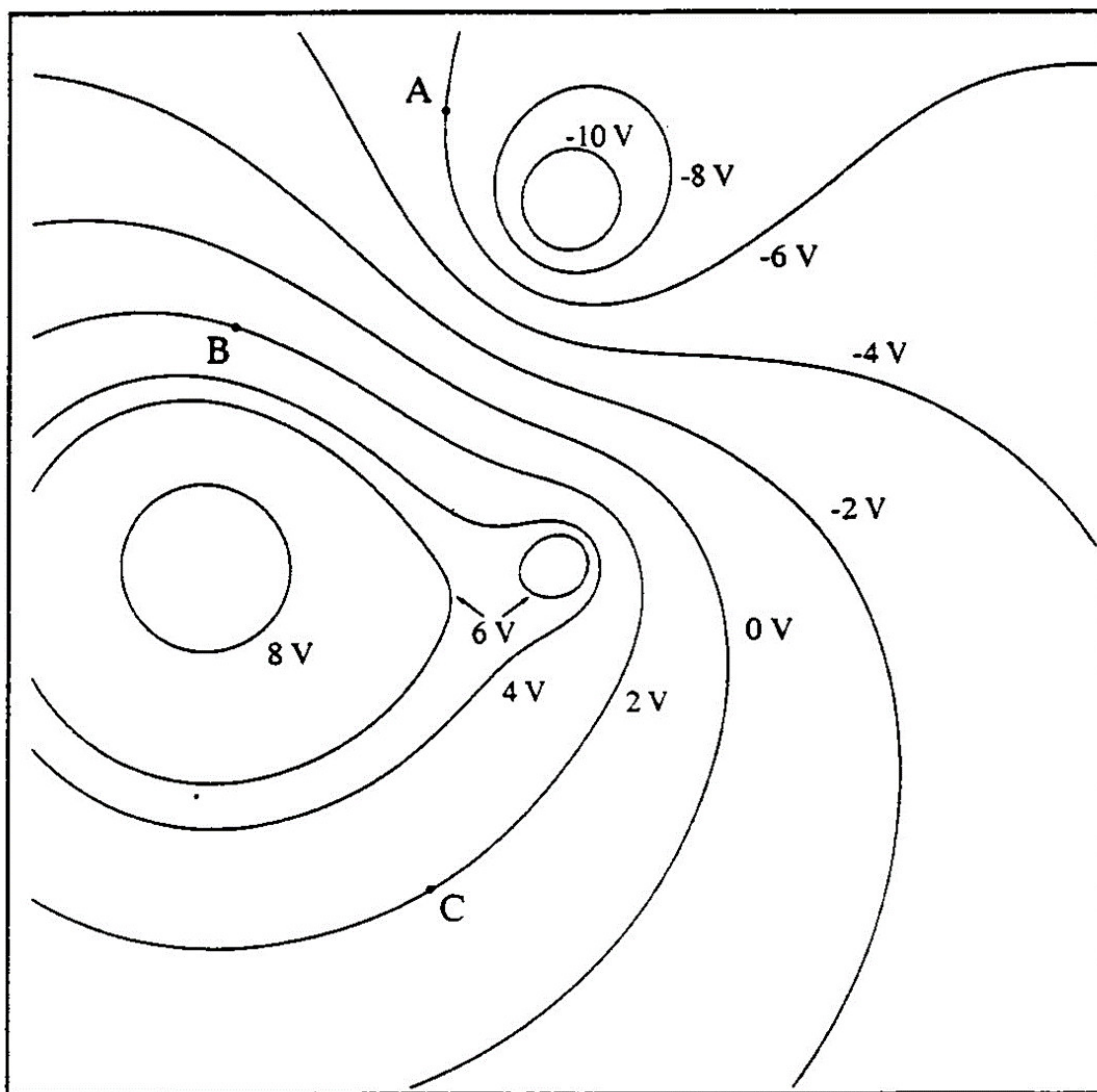
**Q9:** Let's make it interesting. Change your charge distribution so that there are three times as much positive charge as negative charge. Draw the new electric field lines and equipotentials below. Use PhET to help you visualize.



In above two exercises, you drew electric field lines and equipotentials from a known charge distribution. But the truth is, in experimental setup, it is usually difficult to directly measure charge distribution or electric fields. In fact, it is easiest to measure the voltage (to trace out equipotentials)

and infer electric field and charge distribution from that. Use the exercise below to simulate what this would be like.

**Q10:** Shown below is a possible equipotential lines from a possible two-dimensional configuration of charges. Draw, in the figure below: (i) electric field lines consistent with the shown equipotentials, and (ii) possible distribution of charges resulting in below equipotentials. Use PhET to verify your guesses.



*Equipotential lines from UC Berkeley Physics 7B electrostatics worksheet*

**Q11:** Referring to Q10, Answer the following two short questions. Be sure to explain your answers:

- (i) At which of the three points, A, B, or C, is voltage farthest from 0 V (the voltage at infinity)?
- (ii) At which of the three points, A, B, or C, is the electric field strongest?

**Q12:** *[Extra/Optional]* If there is time, set up charge distributions in PhET simulation which produce equipotentials similar to that given in Q10. When you have set it up, call me to verify it and initial here. [Note: This question is optional, as it could take a significant amount of time.]

# Lab: Introduction to Circuits Manual

*Note: For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on separate pieces of paper to turn in. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.*

*This lab is based on UC Berkeley Physics 7B Lab, "Introduction to DC Circuits"*

## Introduction

So far, we have been dealing with electric charges in static equilibrium ("electrostatics"). With this lab, we now consider situations where charges are in motion, which results in **current** (electrical current is measured the same way flow rate of a fluid is measured: amount of charges flowing through a cross-sectional area per second).

The focus of this lab is on *conceptual understanding* of electric circuits. While it is O.K. to know electric circuit formulas (such as addition of resistances in parallel or series), try to explain your answers in this lab conceptually.

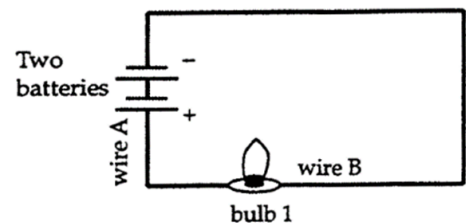
Parts A and B of the lab below will refer to Circuits 1 through 4, diagrammed on the right. Some notes:

- The two batteries in series can be thought of as a single battery with double the voltage of one battery.
- I will demonstrate:
  - How to measure voltages and currents in a circuit
  - How to "transform" one circuit into another

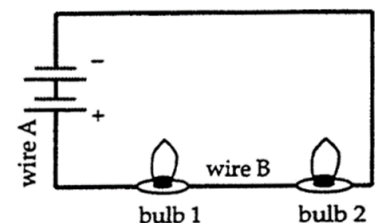
Ask for help if you are having trouble with either (or anything else, really).

- Answer the questions in order. First do the prediction, and then form your final answer based on your observation and understanding. (You are only graded on your final answer; no need to bother changing predictions.)

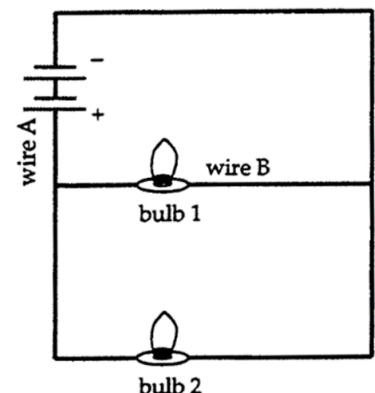
**CIRCUIT 1**



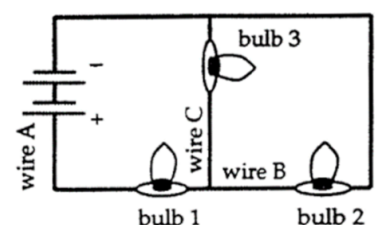
**CIRCUIT 2**



**CIRCUIT 3**



**CIRCUIT 4**



## Part A: Circuits 1 through 3

**Q1:** In Circuit 1, which (if either) is bigger: The current through wire A or the current through wire B? In your lab report, first give your **prediction** of the result, explaining your reasoning. Make the measurement and write down what happened. If it didn't agree with the prediction, make sure to **explain why** what you see happened (if it agrees with the prediction, we presume your explanation for the prediction was correct).

**Q2:** What gets “used up” when current flows through a light bulb?

**Q3:** When circuit 1 is transformed into circuit 2, what happens to: (a) The brightness of light bulb 1? (b) The current through wire A? In your lab report, first give your **prediction** of the result, explaining your reasoning. Make the measurement and write down what happened. If it didn't agree with the prediction, make sure to **explain why** what you see happened (if it agrees with the prediction, we presume your explanation for the prediction was correct).

**Q4:** When circuit 1 is transformed into circuit 3, what happens to: (a) The brightness of bulb 1? (b) The current through wire A? (c) The current through wire B? In your lab report, first give your **prediction** of the result, explaining your reasoning. Make the measurement and write down what happened. If it didn't agree with the prediction, make sure to **explain why** what you see happened (if it agrees with the prediction, we presume your explanation for the prediction was correct).

## Part B: Circuits 4 and Additional Questions

Before you proceed, make sure you got Q1 through Q4 correct. We will go through the first three questions as a class; if you are working ahead, check with me before you proceed.

**Q5:** When circuit 2 is transformed into circuit 4 (by connecting bulb 3), what happens to: (a) The current through wire A? (b) The brightness of bulb 1? (c) The brightness of bulb 2? In your lab report, first give your **prediction** of the result, explaining your reasoning. Make the measurement and write down what happened. If it didn't agree with the prediction, make sure to **explain why** what you see happened (if it agrees with the prediction, we presume your explanation for the prediction was correct).

Make sure your explanations are in terms of **intuitive arguments** (avoid using resistance addition formulas). When you have discussed your explanations in Q5 within your group, call me to get my initial on your lab report. **Lab reports missing instructor's initial will lose points.**

**Q6:** Let  $I_1$  be current through wire A in circuit 1. In terms of  $I_1$ , what is the current through wire A in: (a) Circuit 2? Is it  $2I_1$ ,  $I_1/2$ , or some other value? Explain your answer conceptually. (b) Circuit 3? Explain (same as with (a) again). (c) Circuit 4? You will need formulas we probably did not yet cover

in class, so use conceptual reasoning to give a lower and upper bound for the current (what is the smallest you think the answer can be, and the biggest you think the answer can be?).

**Q7:** With your battery, your three light bulbs, and all the wires you want, build a circuit that produces as much total light as possible. Diagram the circuit in the space provided below, and explain why it's the brightest.

**Q8:** With your battery, your three light bulbs, and all the wires you want, build a circuit that produces as little total light as possible. Should the circuit use all three bulbs? Be sure to test this issue experimentally. Diagram your circuit, and explain why it's the dimmest.

## Part C: Optional Questions

Use the remaining time in lab to answer below optional questions relating to topics covered in this lab.

**Q9:** In this lab, you have built a total of six circuits: the circuits 1 through 4, the “brightest” circuit from Q6, and the “dimmest” circuit from Q7. Answer and explain (how you know): (a) Of those six circuits, which one has the most current flowing through the battery? (b) Of those six circuits, which one has the least current flowing through the battery?

**Q10:** Give at least two separate reasons why it's advantageous to wire winter holiday lights (pictured below) in parallel (i.e. like circuit 3).



**Q11:** Are the electrical outlets in your house wired in series (i.e. like circuit 2) or in parallel (i.e. like circuit 3)? Explain.

**Q12:** In electrostatics, we defined voltage as following,  $\Delta V_{a \rightarrow b} = - \int_a^b \vec{E} \cdot d\vec{x}$ , meaning a voltage difference always corresponds to a non-zero electric field. Is this also true about circuits? Specifically, does the potential difference between the two terminals of the battery correspond to an electric field anywhere? Or do circuits allow us to have “voltages without fields”?



# Worksheet Lab: Circuit Analysis Manual

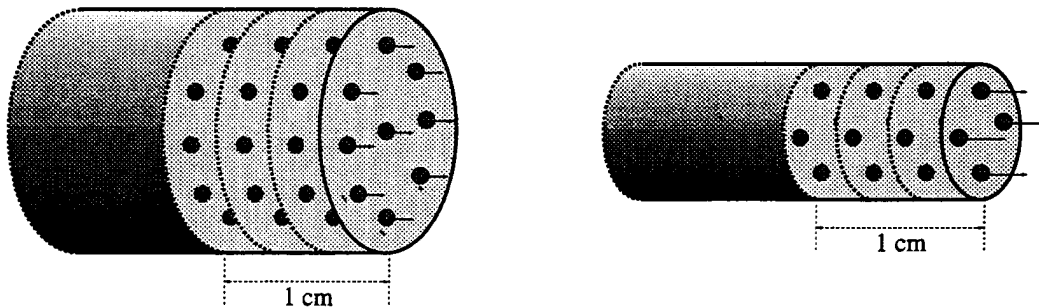


*Note: For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on separate pieces of paper to turn in. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.*

*This lab is based on UC Berkeley Physics 7B Worksheet Lab, "DC Circuits."*

## Questions for Discussion

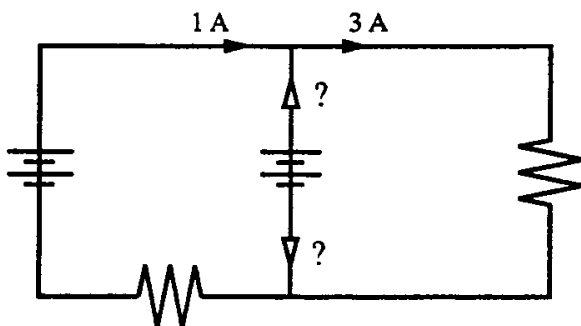
**Q1:** The figure below shows a cutaway view of two current-carrying wires, a thick one and a thin one.



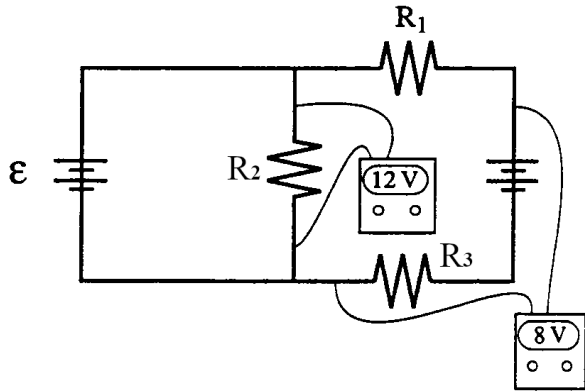
In the thick wire, electrons are moving with a speed of 3 cm/sec. In the thin wire, electrons are moving faster, with a speed of 5 cm/sec.

- First consider the thick wire. In one second, how many electrons will pass a given point in the wire?
- Next consider the thin wire. In one second, how many electrons will pass a given point in this wire?
- Which wire carries the greater current?

**Q2:** In the circuit shown below, what is the value of the missing current? In which direction does it flow?



**Q3:** Voltmeters have been attached to the following circuit, in order to ascertain voltages between various points.

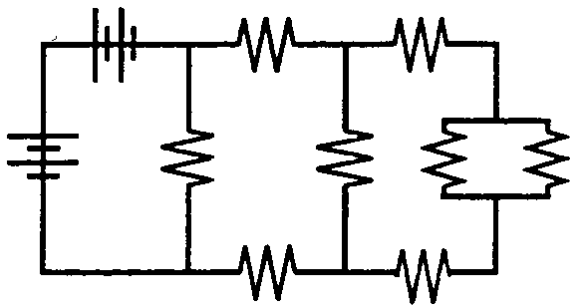


- A. What is the voltage  $\mathcal{E}$  of the battery on the left?  
 B. What is the voltage drop across the resistor  $R_1$ ?

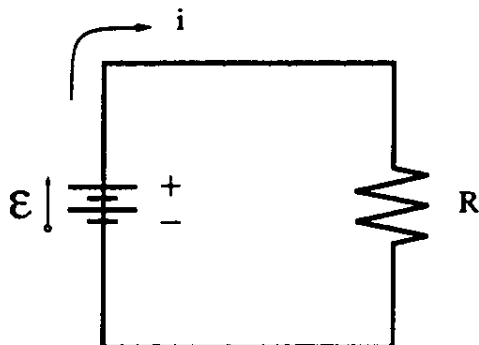
## Problems

*NOTE: In a typical discussion section at UC Berkeley, students usually work through the problems during the discussion section, either individually or in groups. The GSI monitors the progress and conversation among the students and either assists individually or brings the section together for a discussion on a common issue that develops in the course of the section.*

**Q4:** In the following circuit, all batteries are 1V and all resistors are  $1\Omega$ . Reduce the circuit to an equivalent voltage and resistance in series.



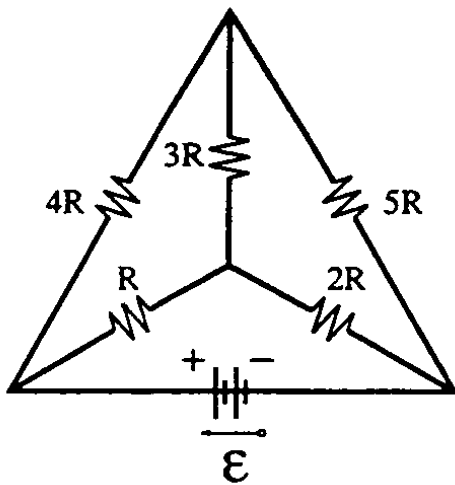
**Q5:** Consider the following simple circuit.






- A. What is the potential difference between one terminal of the battery and the other?
- B. Considering your answer from part (a), how much work does it take to bring an element of charge  $dq$  from the "bottom" (negative) terminal to the "top" (positive) terminal?
- C. Considering your answer from part (b), at what rate is the battery doing work on the charge in the circuit? Answer in terms of the symbols on the diagram.
- D. The current flowing in this circuit is constant in time. What does this imply about the *speed* with which the charge carriers move around the circuit?
- E. Recall that according to the work-kinetic energy theorem, the net work done on the charge carriers must equal the change in their kinetic energy. Considering your answer from part (d), what is the net work done on the charge carriers as they go around the circuit?
- F. We found in part (c) that the battery does *positive work* on the charge carriers as they move around the circuit. So something else must be doing *negative work* on the charges as they move around, in order to satisfy the work-kinetic energy theorem. What element in the circuit is doing negative work on the charges?
- G. In order to satisfy the work-kinetic energy theorem, what must be the rate at which heat is lost through the resistor? Answer in terms of the symbols on the diagram.

**Q6:** Consider the following circuit.

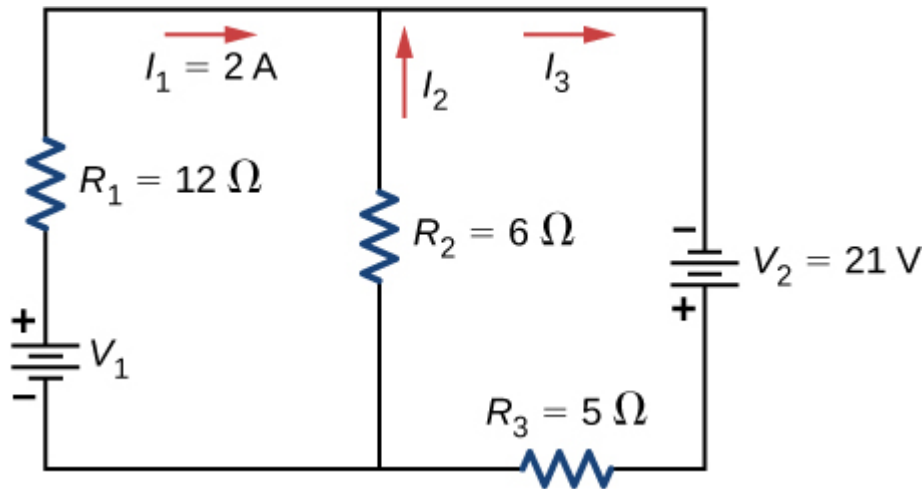


- A. How many distinct currents flow in this circuit?
- B. If we had to solve for these currents, how many independent equations would we need?
- C. On your own diagram, label the 4 junctions and up to 4 loops. Of the junctions you have labeled and loops you have drawn, describe which loops and/or junctions you would choose, in writing down Kirchhoff's rules to solve for the equations. Explain your choice.
- D. Set up a system of equations that would allow you to solve for the currents. (And time-allowing, solve for the currents, either by hand or using a computer algebra system (CAS) such as [SageMath](https://sagecell.sagemath.org/)  [\(https://sagecell.sagemath.org/\)](https://sagecell.sagemath.org/).)

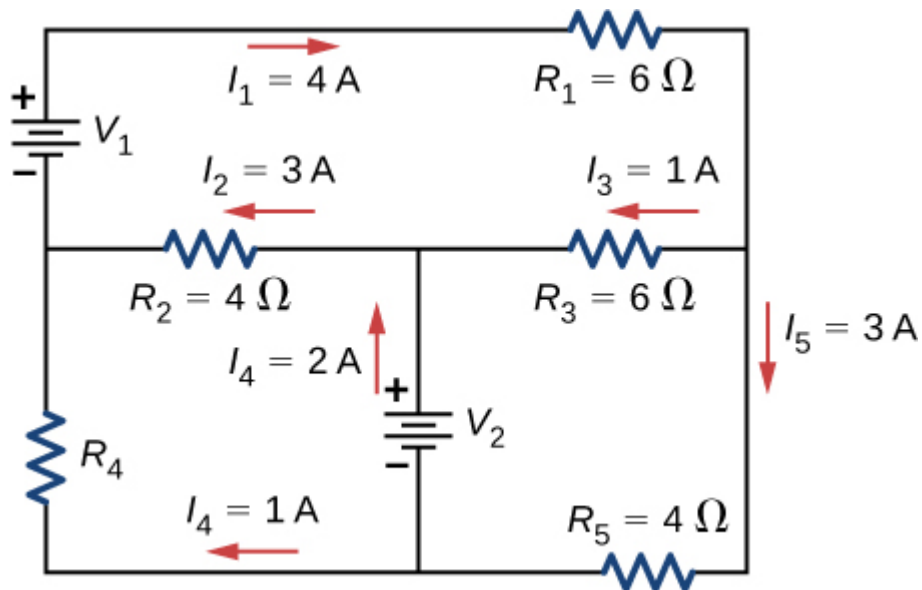
# Additional Kirchhoff Rule Application Exercises

If time allows, answer Questions 38 and 39 from textbook using Kirchhoff's rules (reproduced below):

38. Consider the circuit shown below. Find  $V_1$ ,  $I_2$ , and  $I_3$ .



39. Consider the circuit shown below. Find  $V_1$ ,  $V_2$ , and  $R_4$ .



These are examples of general circuit problems that can *only* be solved using Kirchhoff's rules, so make sure to get some practice solving them.

# Lab: Charge-to-Mass Ratio Manual

*Note: For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on separate pieces of paper to turn in. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.*

*This lab is based on UC Berkeley Physics 7B Lab, "Magnetism Lab: The Charge-to-Mass Ratio of the Electron"*

## Introduction

In this lab, you will explore the motion of a charged particle in a uniform magnetic field, and determine the charge-to-mass ratio ( $e/m$ ) of the electron. We hope that you will also begin to develop an intuitive feel for magnetism.

## Important Background Information about Atoms and This Experiment

All normal matter is made up of atoms. Atoms have a "size" of roughly **1 Å** ("1 angstrom," or  **$10^{-10}$  meter**), and range in mass from about  **$10^{-27}$  kg** to  **$10^{-25}$  kg**. Atoms are in turn made up of smaller particles: positively-charged **protons**, uncharged **neutrons**, and negatively-charged **electrons**. Protons and neutrons have almost the same mass (about  **$10^{-27}$  kg**) while electrons are about  **$10^{-30}$  kg**. Protons and electrons have charge of equal magnitude and opposite sign,  $|q| = e \approx 1.6 \times 10^{-19}$  C.

An atom's protons and neutrons are contained in the atom's **nucleus**, which is about 1 fm ("1 femtometer," "1 fermi," or  **$10^{-15}$  meter**) across—a minuscule fraction of the atom's total size. The number of protons in an atom's nucleus determines what kind of atom it is, where it sits on the periodic table, and its chemical properties. For instance, any atom with six protons is carbon, whereas any atom with seven protons is nitrogen. The number of neutrons in an atom determines which **isotope** of that atom it is. Helium-4 (2 protons + 2 neutrons = 4) has 1 more neutron than helium-3 (2 protons + 1 neutron) and is therefore a different isotope, but both isotopes are still helium atoms because they both have two protons. The study of nuclei is known as **nuclear physics**.

The nucleus is surrounded by the lighter electrons, which take up most of the volume of the atom. A normal atom has the same number of electrons as protons, and so has zero net charge. If the atom has a different number of electrons than protons, it is called an **ion**; ions with more electrons than protons have a net negative charge and are said to be **negatively ionized**, while ions with fewer

electrons than protons have a net positive charge and are said to be **positively ionized**. The study of atoms in general, and their electrons in particular, is known as **atomic physics**.

Understanding the electron is essential for understanding atoms and matter in general. Two important properties of the electron are its charge  $e$  and its mass  $m$ . In this experiment we will measure the ratio of the two ( $e/m$ ) with the method first used by J. J. Thomson in 1897. The experiment is based on the fact that a charged particle moving in a magnetic field feels a force at right angles to its velocity,  $\vec{F}_B = q\vec{v} \times \vec{B}$ . If we send a beam of electrons into a magnetic field uniform in strength and direction, then the trajectory of the electrons is a circle whose radius depends on  $e/m$ . We measure the radius of the circle for different values of  $B$  and deduce  $e/m$ .

In Part A, we will first derive some formulas needed for this lab. In Part B you will make the measurements, paying attention to some common sources of measurement error, and in Part C, you will analyze your measurement to obtain a single value of  $e/m$ .

## Part A: Preliminary Questions; Formula Derivations

**Q1:** Using your Physics 4B knowledge about the force on a charged particle moving in a magnetic field, and your Physics 4A knowledge of circular motion (under a centripetal force), derive an equation for the radius  $r$  of the circular path that the electrons follow in terms of the magnetic field  $B$ , the electrons' velocity  $v$ , charge  $e$ , and mass  $m$ . Please assume that the velocity  $\vec{v}$  of the electrons is perpendicular to the magnetic field  $\vec{B}$  initially. (Aside: What kind of path does the electron follow, if some component of  $\vec{v}$  is parallel to the magnetic field  $\vec{B}$ ?)

**Q2:** Recall from electrostatics (earlier this semester), that an electron obtains kinetic energy when accelerated across a voltage difference  $V$ . Since we can directly measure the accelerating voltage  $V$  in this experiment, but not the electrons' velocity  $v$ , rewrite your equation from Q1 in terms of voltage difference  $V$ , not velocity  $v$  (obtain an expression for velocity  $v$  in terms of accelerating voltage  $V$ , and plug it in), assuming that the electron starts at rest. Solve this equation for  $e/m$  to obtain this result (we will refer to this as Eq. 1 below):

$$\frac{e}{m} = \frac{2V}{B^2 r^2}.$$

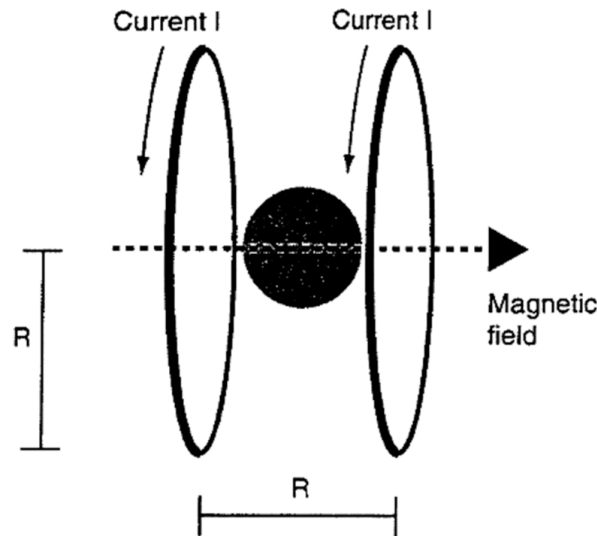
Show your work in your notes.

**Q3:** As derived in lecture, the magnetic field due to a circular loop of current can be calculated along the axis (line going through the center of the circular loop, perpendicular to the plane of the loop). As a function of distance  $z$  away from the loop, the magnitude of the magnetic field is (we will refer to this as Eq. 2 below),

$$B = \frac{2\pi k_e I R^2}{c^2 (R^2 + z^2)^{3/2}} = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}},$$

where  $R$  is the radius of the loop and  $I$  is the current. The direction of the magnetic field along the axis is given by the right-hand rule.

Using this result, calculate the magnetic field at the midpoint along the axis between the centers of the two current loops that make up the Helmholtz coils, in terms of their number of turns  $N$ , current  $I$ , and radius  $R$  (see figure below). Separation distance between the coils in Helmholtz coils is equal to their radius  $R$ .



Show that the magnitude of the magnetic field is given by (we will refer to this as Eq. 3 below),

$$|B| = \left(\frac{4}{5}\right)^{3/2} \frac{4\pi k_e N I}{c^2 R} = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{R}.$$

Show your work in your notes.

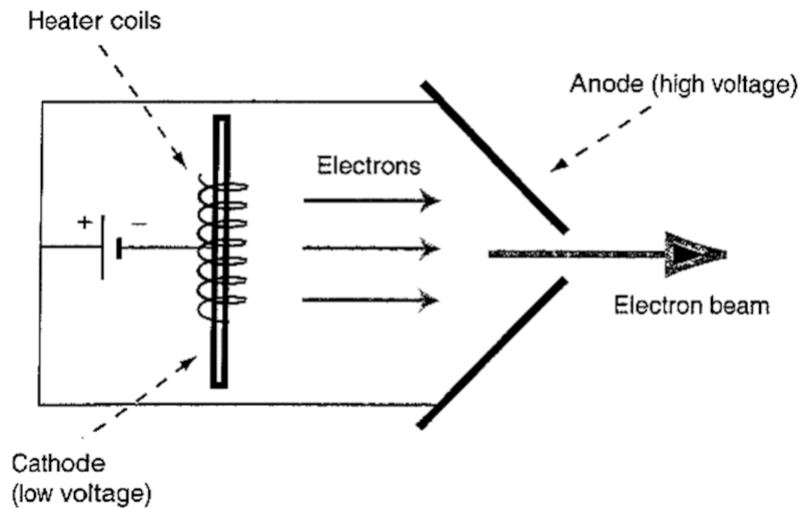
(Aside: The usefulness of Helmholtz configuration comes from the fact that the magnetic field in the center region is very uniform. It can be shown using calculus that the gradient  $\partial B / \partial z$  of the magnetic field at the midpoint is zero (due to symmetric placement of the coils), and the second derivative  $\partial^2 B / \partial z^2$  is also zero (due to the separation  $R$  between the coils).)

## Part B: The Experiment and Measurements

We describe the experimental setup below.

### The Electron Beam

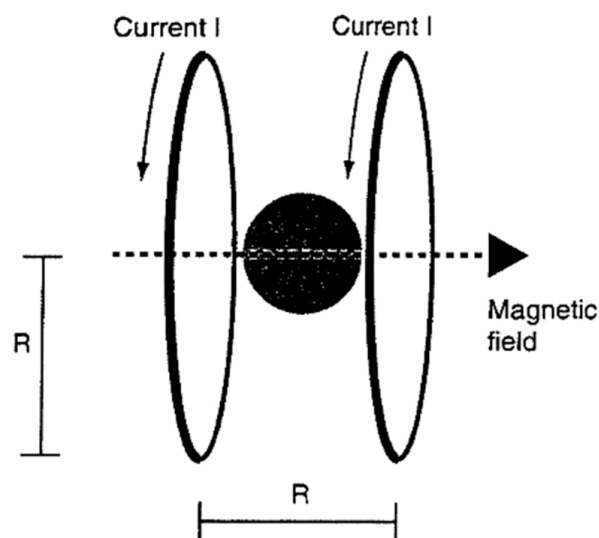
To produce a beam of electrons, we heat a metal plate called a **cathode** and boil electrons off of its surface. The cathode is held at a low voltage (that's why it's called "cathode"), and the boiled-off electrons accelerate towards a high-voltage plate a few centimeters away called an **anode**. Some electrons pass through a small hole in the anode and are collimated into a narrow beam (see figure below).



The speeds of the electrons is roughly constant once they pass through the anode; since we can't see electrons, this apparatus is placed inside an evacuated glass bulb, containing a small amount of helium gas. When the gas molecules are struck by electrons, they radiate a blue color, making the path of the electron beam—but not individual electrons—visible.

## The Magnetic Field

To produce a uniform magnetic field, we place two large circular coils of wire known as **Helmholtz coils** around the tube, one on either side (see figure below, reproduce from above). The two coils have the same radius and the same number of turns (in lab, measure the radius; number of turns is equal to 140 turns) and are placed exactly one radius apart. When a current is passed through both coils in the same direction, the fields add to produce a very uniform magnetic field in the center region between them. The field is pointed along the line joining the centers of the two coils, and its magnitude at the center can be calculated from the current in each coil by Eq. 3 above.



Your experimental equipment is set up so that you can apply accelerating voltage, deflecting voltage, and coil current by simply turning knobs and pressing switches on the front panel of your instrument. Study the front panel and make sure you understand what each knob and switch does; if not, ask

me. For the purpose of this lab, you will not use deflecting voltage, so make sure deflecting voltage switch is off (although, feel free to play around with it later).

Turn all the knobs to the most counter-clockwise position (at zero volts and zero amps). Turn on the small, red power-supply switch for your instrument.

**Q4:** You are going to turn up the accelerating voltage and the electron beam will appear. Will it be curved or straight? Give a prediction (write it down) and explain your prediction.

**Q5:** Now turn up the voltage until you see a glowing blue electron beam. Make your observation and give your final answer (also write it down). If your final answer is different from your prediction, make sure you understand why and write down your explanation, explaining the difference.

Press the coil current switch for clockwise direction of current.

**Q6:** You are going to turn up the coil current and the electron beam will form into a circular shape. How will the radius of this circular beam path change (will it increase or decrease?): (i) if you increase the magnetic field? (ii) if you increase the accelerating voltage? Give a prediction (write it down) and explain your reasoning *conceptually* (that is, without simply referring to Eq. 1; explain from first principles).

**Q7:** Try out each change in Q6 above, and give your final answer (also write it down). If you cannot increase accelerating voltage any higher, try decreasing them (and adjust your answers accordingly). Make sure to give an explanation, if your final answer is different from prediction.

Set the voltage and magnetic field so that you see a well-defined circular electron beam path. You will be measuring the radius of the path, but first read about parallax error below and develop a radius-measuring technique to avoid the error. [Hint: Note the rulers attached to the frame for the Helmholtz coils and the strip of mirror on the other side.]

### Parallax Errors

Close one eye and hold up a ruler between you and a far wall. Now, move the ruler towards or away from your eye without moving your head, so that the ruler just covers the wall from end to end. If you didn't know better, you'd think that you had just measured the length of the wall to be the same as that of the ruler. This is an example of **parallax error**, which can occur when a measuring stick is not placed directly against the object it is measuring. This is the same reason why you are told to read thermometers and graduated cylinder readings (if you took chemistry) at eye-level, so that you see the liquid level correctly aligned to the measuring marks.

To avoid parallax error, you have to develop a method to correctly position your eye.

**Q8:** Explain your method of avoiding parallax errors. Explain how it works. [Note: I do want you to think about this a while to try to figure it out, but I do not want you to waste too much time doing this.

As a group, think and discuss for about 10 minutes, and if you are not sure, call me so that we can go over the measurement techniques.]

**Q9:** Now, measure the radius of the electrons' path. Record and organize your data in your notes. Your data should consist of following numbers: (1) radius of the beam path (what you just measured), and (2) relevant independent variables (they are: accelerating voltage and coil current). Repeat the measurement for at least four other values of independent variables (Helmholtz coil current and accelerating voltage combinations). [*Give some consideration: if two different combinations of Helmholtz coil current and accelerating voltage result in the same radius of beam path, do these constitute two different measurements or are they one, single unique measurement?*]

You may want to organize your measurements as a table, in order to clearly keep track of your measurements and calculated values of  $e/m$  (see analysis questions on next section).

## Part C: Analysis Questions

**Q10:** Calculate the average value of  $e/m$  from your measurements, and compare it to the accepted value of  $1.76 \times 10^{11} \text{ C/kg}$ , by computing percent error.

**Q11:** One possible source of error is the difference between your calculated value of magnetic field and the actual magnetic field in the experimental region. We have digital magnetometers that can be used to directly measure the value of the magnetic field. Call me to help you get set up with the magnetometer. Once you are set up, measure the magnetic fields where you can; compare the values (measurement and calculated) of magnetic fields. Does this explain a portion of your error in Q10 above?

The question below is for the errors that remain *after* correcting for any differences in magnetic field.

**Q12:** Does your result appear to be dominated by **statistical error** (that is, error varying from measurement to measurement and attributable to lack of precision and random factors) or by **systematic error** (that is, error resulting from an equipment calibration error or other sources that affect all results systematically)? Give an explanation using your data and results from Q9, Q10, and Q11. Write down possible sources of error, and where possible, estimate the amount of uncertainty (in percent error terms) attributed to each source.



Your Name:

Group Partners' Names:

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## Lab 7 – Electric Motor

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### Introduction

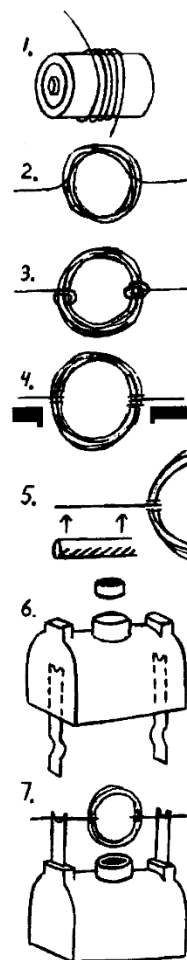
In this lab, you will build and observe the working of two simple forms of motor. Build them following the instructions in the next section, and answer the simple four questions in the next section.

### Part A: “World’s Simplest Motor” and Homopolar Motor

#### World’s Simplest Motor

A kit is provided for this motor. Please follow directions below to build a working motor:

1. Unwrap the wire and straighten out any bends. Leaving about 2 inches straight, (about the length of a D-cell battery), wrap the wire around the battery to form a coil (Figure 1). Unwrap a small amount from the 2<sup>nd</sup> end so that you now have about two inches of wire sticking out from either side (Figure 2).
2. Each end of the wire is wrapped tightly around the coil for two turns (Figure 3). This will keep the coil together. The two ends should stick out directly opposite of each other and should be at least 1 inch long. Excess can be trimmed or wrapped around the coil as additional turns.
3. The wire is covered with an enamel coating for insulation. Hold the coil vertically and then rest one of the wire ends on a flat surface (Figure 4). Using the edge of a metal support, **scrape the enamel coating of the entire top half of the wire end**. Turn the coil slightly as you scrape so that the **top half** of the wire is scraped bare. *Do not scrape the bottom half of the wire*. Repeat this for the 2<sup>nd</sup> wire sticking out from the opposite end of the coil. **The enamel is left on the bottom half of each wire** (Figure 5).
4. Slide the metal supports (U-end first) up through the slots in the plastic base. The bump in the metal faces towards the battery. The battery is pushed in and must touch both metal supports (Figure 6). Set the magnet into the round holder.
5. Set the coil ends into the U of the supports and your motor is ready to run. Give the armature a gentle spin (Figure 7). If it does not continue to turn, try the opposite direction. After the motor works, the very ends of the wire can be bent to help keep the coil centered. If the motor does not work, check to see if the shiny side of the wires are both facing up when the coil is vertical (Figure 5). Also, make sure the wire ends of the coil are centered (Figure 3).



#### Homopolar Motor

Using the provided materials, assemble AA battery, neodymium magnet (and washers for mechanical support), and uncoated copper wires into the arrangement shown in the picture on left, and see the wire spin as a motor (make sure the wire makes electrical contacts at the top and bottom).

(Questions on Next Page)

## Part B: Questions

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Please answer the questions below using the space on the page. Attach additional pages if needed.

- Q1:** How does the “World’s Simplest Motor” work? Describe the torque due to the magnetic field on the wire loop as it turns. Explain why it is necessary to ensure only one side of the wire is scraped of enamel to make electrical contact. [Note: Try making a motor with wire fully scraped of enamel all around. Does it still work? If it does, any explanation why it still works in practice?]
- Q2:** How does the homopolar motor work? Draw a diagram to illustrate the direction of current flow, magnetic fields, and the force on the wire. Explain why it is not necessary to switch the polarity of current (hence “homopolar”) on this motor.
- Q3:** Measure the frequency of rotation of “World’s Simplest Motor” you built using an oscilloscope (measure the voltage of the battery with the oscilloscope as the motor runs; there should be some periodic, small change in voltage; call me if you need help setting up oscilloscope to measure the frequency of rotation). What parameters limit the frequency of the motor? List 3 things you might do, if you want the motor to run faster.
- Q4:** If you look at the motor carefully (either “World’s Simplest Motor” or the homopolar motor), what you have done is short the two ends of battery (the one thing I kept telling you not to do for safety reasons). But it turns out, as long as the motor is running, this is safe: the current output from the battery is limited, and the battery will not overheat even in extended operation. Explain the physical mechanism that limits the amount of current flow in the motor wires (this is somewhat related to Q3).

# Lab: Introduction to Oscilloscope Manual



*Note: For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on separate pieces of paper to turn in. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.*

## Introduction

Oscilloscope is an indispensable electronics laboratory instrument. At its core, it is a voltmeter which can display its measurement as a function of time. But this basic functionality can be used to perform a wide range of measurements and tests.

In this lab, we will introduce basic functionalities of the oscilloscope and its use in analyzing some simple DC circuits.

## Part A: "Initial AC Operation"

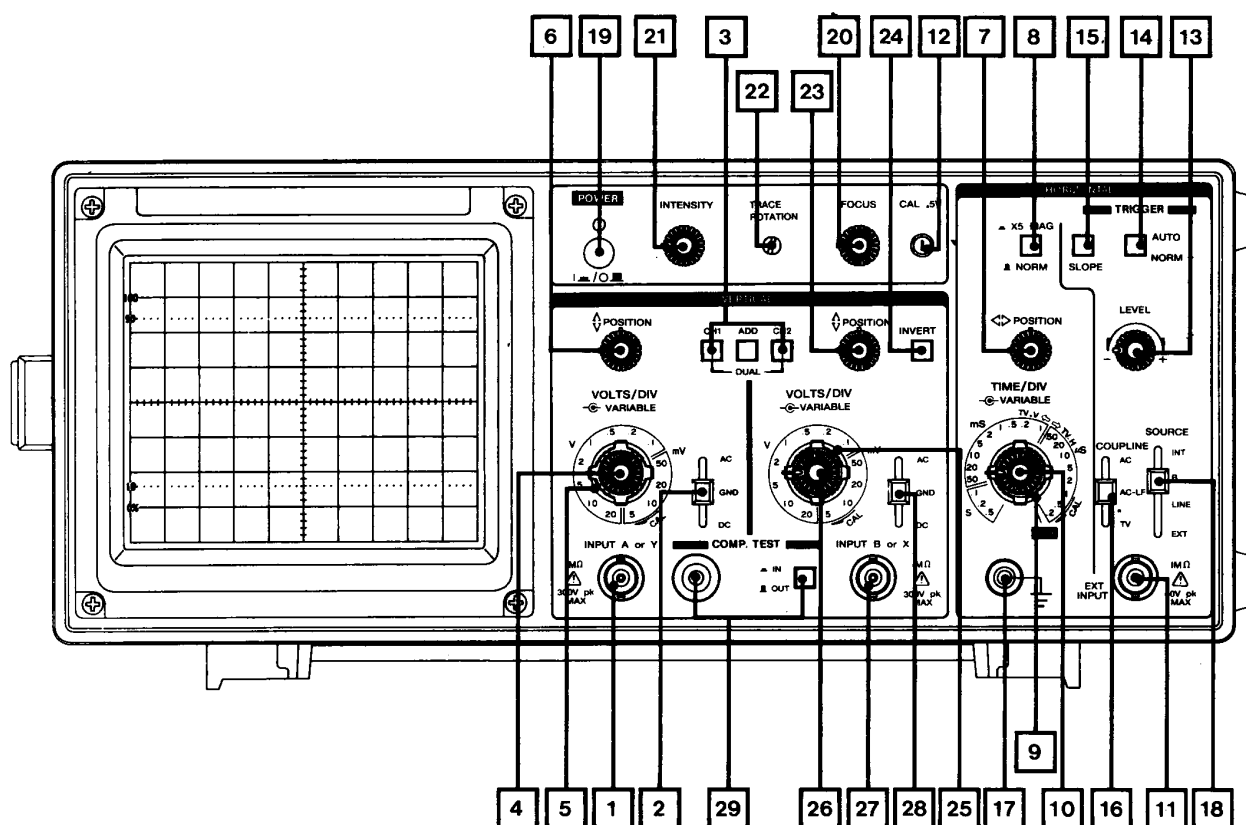
One of the most intimidating thing about the oscilloscope is the sheer number of controls (digital oscilloscopes are sometimes better, because there can be a menu to scroll through, but it will always be the case that oscilloscopes are highly customizable, with a complex user interface), and for each oscilloscope (never mind the very first one you use), it will take some time to learn its interface. On *some* oscilloscopes for some people, it could take a while to find the power button.

Our goal in this lab is not to make you an expert in oscilloscope configuration and use (it takes more than one lab period), but you will be able to make some basic measurements. This part is the first step in oscilloscope use. Follow the initial set up instructions below to get started. (*Note: These instructions are from the [oscilloscope manual, attached as an appendix](https://peralta.instructure.com/courses/46578/files/4200211?wrap=1) (<https://peralta.instructure.com/courses/46578/files/4200211?wrap=1>) to this lab. My notes—including which parts you can skip—will be in italics.*)

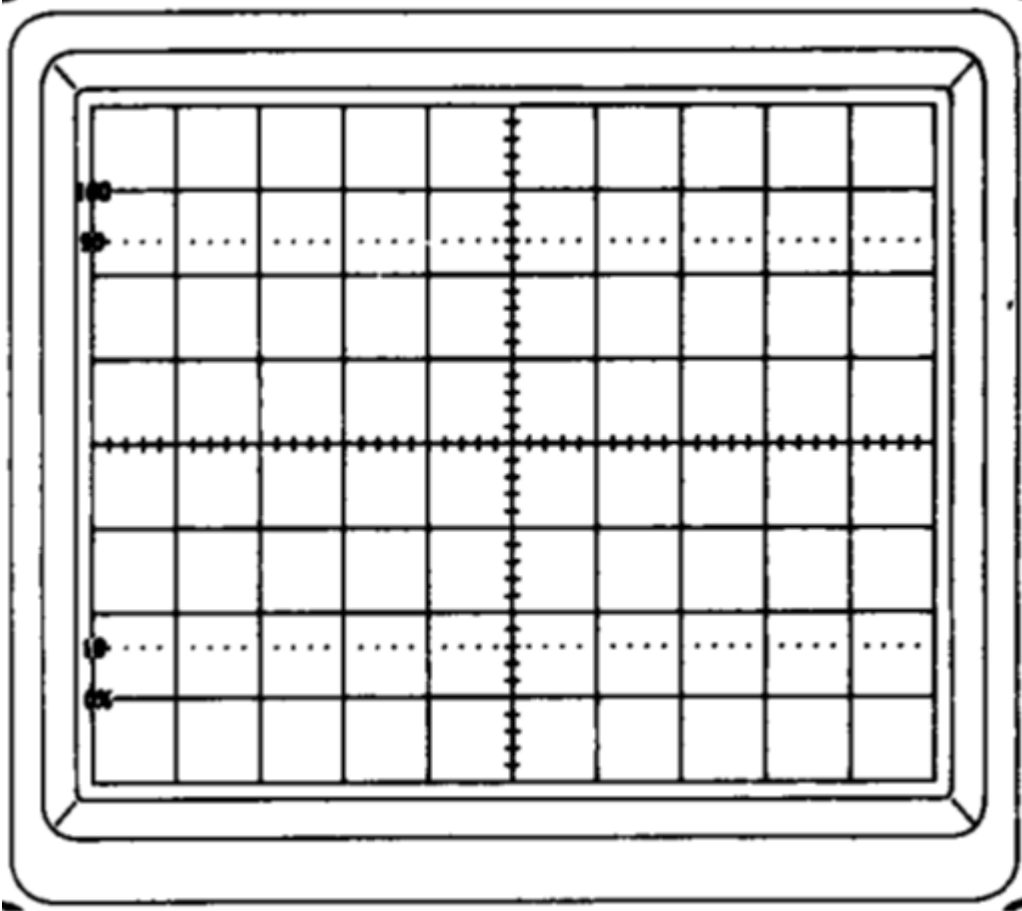
## From pg. 2, Section 3-1 of P-3502C oscilloscope manual

1. Prior to any kind of operation of the instrument, proceed as follows to get familiarized with the instrument. (*Diagram of front panel with labels for controls is on the next page.*)
  - a. Set the POWER switch to OFF.
  - b. Turn all the three POSITION controls to mid-position.
  - c. Turn INTENSITY control to mid-position.

- d. Push TRIGGERING LEVEL control for AUTO.
  - e. The rest of the controls remain at any position for normal operation.
  - f. Check the line voltage. *(No need; the "line voltage" in the lab is 110 volts, like anywhere else in U.S.)*
2. Connect the AC line cable into the AC receptacle on the rear panel of the instrument, and plug into an AC power outlet. *(No need; already done for you when the lab was setup.)*
  3. Turn POWER to ON. After approximately 20 seconds, trace lines appear on CRT screen. If no trace lines appear, rotate INTENSITY clockwise till trace lines are easily observed.
  4. Adjust FOCUS and INTENSITY controls for clear trace lines.
  5. Readjust VERTICAL and HORIZONTAL POSITION controls for locations required.
  6. Connect a probe (10:1) to INPUT of CH-A and hook the tip of the probe to CAL 0.5Vp-p output. *(I recommend that you use the probe at 1:1 setting. 10:1 setting will make the signal you see smaller. CAL output is located next to the FOCUS knob.)*
  7. Rotate CH-A Vertical Attenuator VOLTS/DIV switch to 10mV/DIV and turn the VARIABLE on the same axis clock-wise to detent. Turn TRIGGERING SOURCE to CH-A. Then a square wave of 5 divisions is displayed on the screen. *(With a 1:1 probe, you should turn VOLTS/DIV switch to 100mV/DIV to see a 5-division square wave. Turning the VARIABLE knob all the way clockwise makes sure that your reading is calibrated.)*
  8. If the square-wave is distorted, adjust the trimmer of the probe till it becomes a good square-wave. *(Call me if adjusting the trimmer is necessary. In vast majority of cases, it should not be necessary.)*
  9. Remove the probe tip from CAL 0.5Vp-p output. Now, the oscilloscope is ready for use. *(Answer the questions below before removing the probe tip.)*



**Q1:** Turn SWEEP TIME/DIV (#9 control on the diagram of oscilloscope) until you get several periods of the square wave on the screen. Sketch the square wave in the space below.



**Q2:** Measure the frequency and period of the square wave you see. Describe in your notes how you measured it and your measurement. [Hint: pg. 7 of the equipment manual gives a description of how different parameters of a sine wave can be measured, which may be relevant here.]

Now your oscilloscope is set up to a reasonable initial setting and you are ready for the rest of the lab.

## Part B: Exercises with Function Generator

A function generator is a device that can produce a time-varying voltage signal. A standard function generator can produce at least sinusoidal waves, triangular waves, and square waves (higher-end function generator can generate a user-defined “arbitrary” periodic function, or even a single pulse of signal). In this part, you will use a function generator in combination with the oscilloscope to explore its behavior.

Take a look at the function generator interface. Use the frequency controls (combination of RANGE buttons and COARSE and FINE knobs) to set the output frequency at 1 kHz. Select the sinusoidal function button. Connect the OUTPUT of the function generator to CH-A of oscilloscope and change

OUTPUT LEVEL until you see a 1 Vp-p sine wave (1 volt “peak-to-peak,” meaning the voltage difference from the highest point on the sine wave to the lowest point is 1 volt). Adjust SWEEP TIME/DIV knob so that you see several periods of sine wave.

**When you have 1 Vp-p sine wave displayed on your oscilloscope, call me to get this section initialed. Lab write-ups without my initial will lose points.**

(Also call me if you are having any difficulty getting this to work.)

**Q3:** Select the triangular function button. Does what you see on the oscilloscope make sense? Explain if not. (This is not a trick question.)

Make sure that AC-GND-DC control is on “AC” for CH-A. Then, change the frequency of the triangular wave to 100 Hz, then 10 Hz, and then 1 Hz. Each time, adjust SWEEP TIME/DIV knob so that you continue to see several periods of the triangular wave. If possible, try frequencies less than 1 Hz.

**Q4:** Do you see anything strange or unexpected (about the function shape)? Describe in your notes.

**Q5:** Try above exercises with square wave. Again, starting from 1 kHz frequency and adjusting SWEEP TIME/DIV knob to display several periods of the square wave. Describe below anything you see that could be described as strange or unexpected (perhaps you expected it after Q4).

**Q6:** For one last time, repeat the exercises in Q5, but after switching AC-GND-DC control to “DC” for CH-A. What do you see? Do you see anything strange or unexpected?

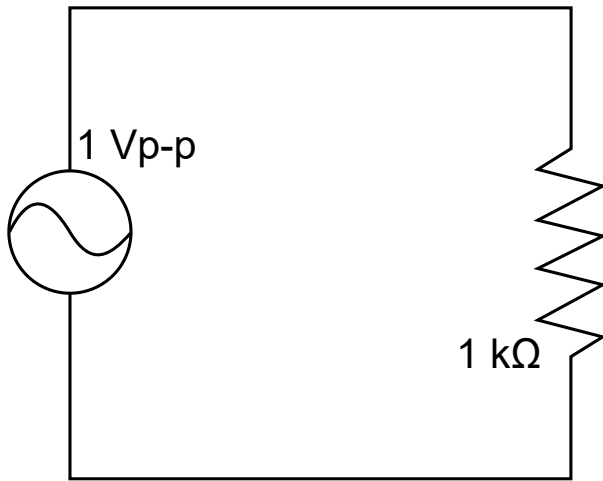
Above exercises is the reason why we recommend the following: “With your oscilloscope, always use the DC mode, unless you know *why* you should be using AC mode for a particular measurement.” This advice applies even when you expect an AC signal. (One of the valid reasons for using AC mode is to remove a particularly large DC offset, if you know that your AC signal is at a reasonable frequency.)

## Part C: Voltage Divider Circuit

Let’s use the oscilloscope to make at least a few real measurements.

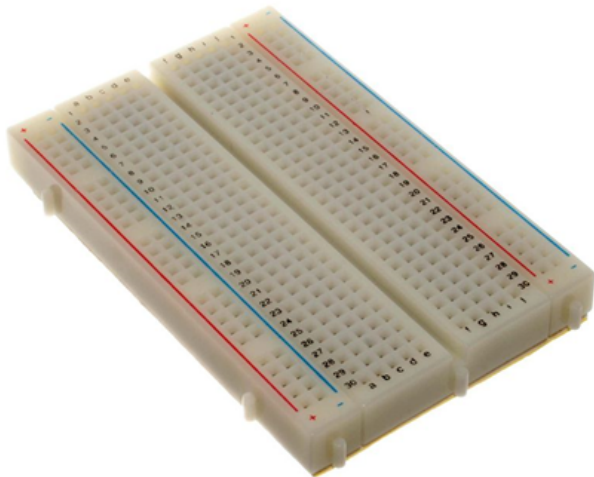
**Q7:** Take four 1 k $\Omega$  resistors available in the lab. Use the provided DMM (digital multimeter) to measure the resistance of the resistor. Write down their resistances in your notes. Keep the resistors separate so that you know which resistor has which resistance (they vary a little bit from each other).

“Build” the following circuit below. You may choose to use the breadboard provided. (More description of breadboard is below before Q9, where you’ll need to use it.)

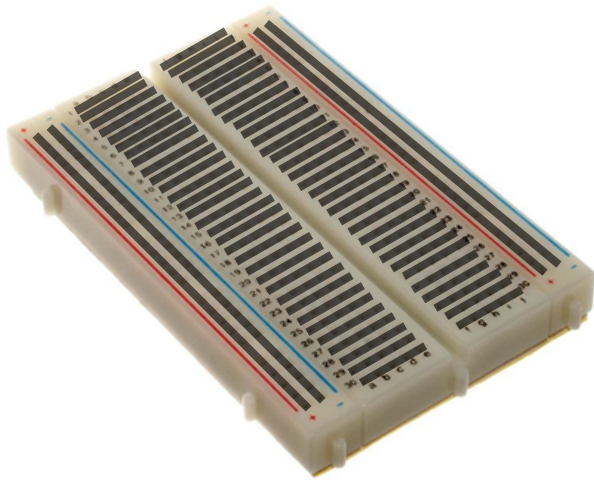


**Q8:** Considering the circuit you just built (diagrammed above), come up with a way to measure the current through the resistor using the oscilloscope. [Hint: It's not complicated, but it will involve some measurement and some *calculation*.] Record the current through the resistor.

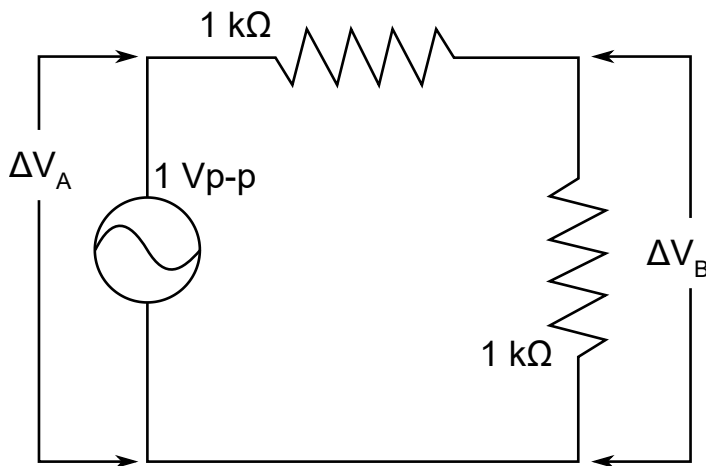
Shown below is a picture of the breadboard.



A breadboard allows for a quick and easy construction of circuits (for prototyping purposes, usually) by providing multiple terminals which are electrically connected inside the breadboard. For the breadboard shown above, the terminals which are connected electrically (with close to zero-ohm resistance) are shown connected on the diagram below, with gray bar over the connected terminals (the difference between orientation is the difference between the “power bus” and regular connections).



Using the provided breadboard, build the following simple circuit. Use the two channels on the oscilloscope to measure  $\Delta V_A$  and  $\Delta V_B$ . [Caution: The oscilloscope input is always grounded. Make sure the ground connections for all instruments touch the same point on the circuit to make sure you don't accidentally short-circuit some part of the circuit.]



**Q9:** Measure  $\Delta V_A$  and  $\Delta V_B$ . What is the ratio  $\Delta V_B / \Delta V_A$ ?

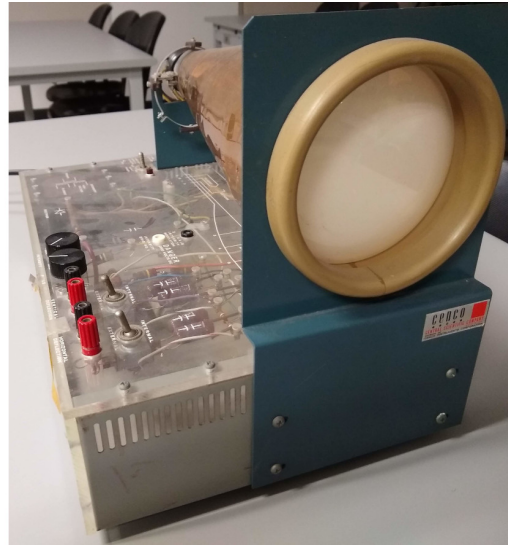
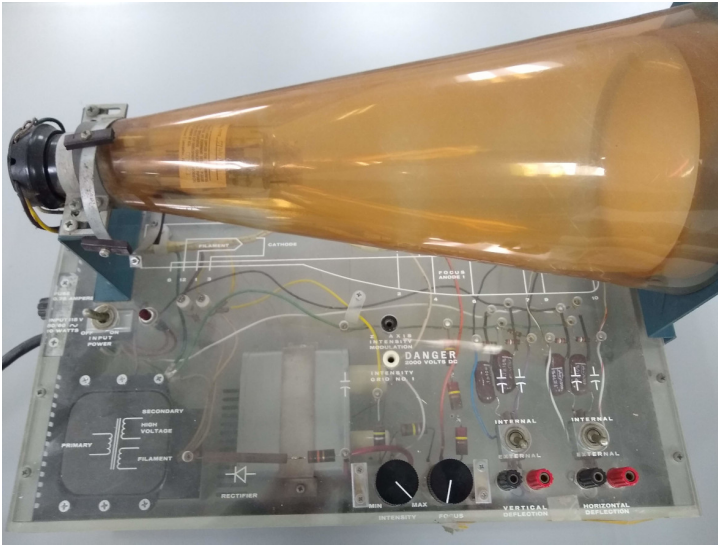
Modify the circuit by increasing the resistance of the top resistor up to 3 kΩ (put three 1 kΩ resistors in series; call me for help if you are not sure how to do that on the breadboard).

**Q10:** Measure  $\Delta V_A$  and  $\Delta V_B$ . What is the ratio  $\Delta V_B / \Delta V_A$  for the new circuit?

**Q11:** Guess the formula for  $\Delta V_B / \Delta V_A$  in terms of the resistances  $R_1$  and  $R_2$  (let  $R_1$  be the resistor on the top, and let  $R_2$  be the resistor on the right). Prove that this is the correct formula by analyzing the circuit (using the tools you learned in lecture; call me if stuck). Can you explain why this circuit is called “voltage divider”? [Note: This is the template for *several* different circuits you will see this semester, once we introduce RC, LR, and LRC circuits. Learning how to analyze this now will be helpful later.]



## Extra: Demonstration Oscilloscope (and effect of magnetic force)



We have a demonstration oscilloscope with mostly transparent compartments, so that you can see the working parts of the oscilloscope. Take a look and see if you can identify each part. Also, bring a strong magnet (provided; call me if you don't see it) near the oscilloscope while it is operating and try if you can see the deflection of the electron beam which traces out the oscilloscope picture.

## Appendix

Oscilloscope manual from the manufacturer is provided in link below.

[P3502-manual-operations.pdf \(https://peralta.instructure.com/courses/46578/files/4200211?wrap=1\)](https://peralta.instructure.com/courses/46578/files/4200211?wrap=1)



# Lab: Time-Dependent Circuits Manual

*Note: For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on separate pieces of paper to turn in. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.*

## Introduction

With introduction of Faraday's Law and inductors (whose working can be explained by electromagnetic induction through Faraday's Law), you have now seen all the circuit elements that are collectively referred to as "linear circuit elements". They are: resistors, capacitors, and inductors. A brief summary table describing their properties in terms of resistance (**R**), capacitance (**C**), and inductance (**L**) is below.

### Resistors ()

Ohm's law:  $I = V/R$

voltage change across a resistor:  $V = IR$

power dissipated in a resistor:  $P = I^2 R$

addition in series:  $R_{eq} = R_1 + R_2$

addition in parallel:  $1/R_{eq} = 1/R_1 + 1/R_2$

### Capacitors ()

definition of capacitance:  $C = Q/V$

voltage change across a capacitor:  $V = Q/C$

current through a capacitor:  $I = dQ/dt$

energy stored in a capacitor:  $E = Q^2/2C$

addition in series:  $1/C_{eq} = 1/C_1 + 1/C_2$

addition in parallel:  $C_{eq} = C_1 + C_2$

### Inductors ()

definition of (self) inductance:  $L = V/(dI/dt)$

voltage change across an inductor:  $V = L(dI/dt)$

energy stored in an inductor:  $E = LI^2/2$

addition in series:  $L_{eq} = L_1 + L_2$

addition in parallel:  $1/L_{eq} = 1/L_1 + 1/L_2$

With the introduction of capacitors and inductance—and methods to analyze them in a circuit—now we can analyze the behaviors of time-dependent circuits (in the past, you have looked at the limited cases of a capacitor circuit, either *immediately* after you change the voltage input or a *very long* time after), as well as where an oscillating voltage is applied (called "AC circuits").

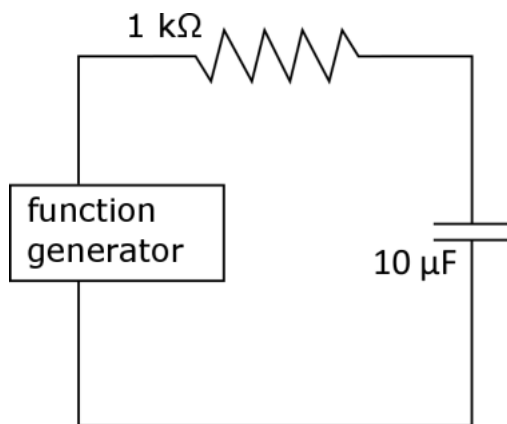
Because this covers a rich set of topics that can both be tackled at a conceptual level and through rigorous mathematics (solving differential equations), I recommend doing some preparation work before the lab, in order to ensure that you can use your 3 hours in the lab as efficiently as possible. If you are already at the lab, please complete below preparation work as quickly as you can (no more than 30 minutes), and call me if it's taking longer than 15 minutes.

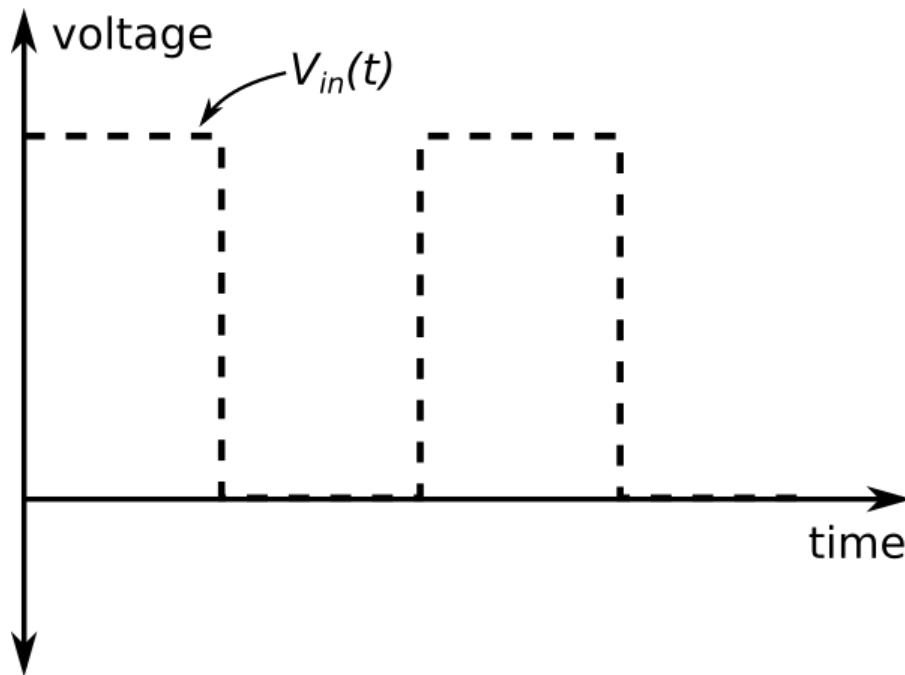
## Lab Preparation: Oscilloscope and Topics Review

Before or at the beginning of the lab, please complete below tasks and answer Q1 below:

**Oscilloscope Intro:** We introduced the oscilloscope in a previous lab ([Lab 10: Introduction to Oscilloscope Manual](https://peralta.instructure.com/courses/46578/pages/lab-10-introduction-to-oscilloscope-manual) (<https://peralta.instructure.com/courses/46578/pages/lab-10-introduction-to-oscilloscope-manual>)). Review the lab, particularly Part B, displaying a 1-kHz signal and looking at waveforms of different shape generated by the function generator. If necessary, take the time at the beginning of this lab to show 1 kHz signal on your oscilloscope (and call me if any portion of oscilloscope operation doesn't make sense).

**Q1:** Consider the RC circuit shown below, which is being driven by a function generator supplying a voltage of  $V_{in}(t)$ , shown in the figure below the circuit. Sketch the voltage that would be measured across the 10 microfarad capacitor. Assume that the period of the input square wave is several times longer than the time constant  $RC$ . (If these instructions do not make sense, watch the lectures in [Lecture: Analysis of RC and LR Circuits](https://peralta.instructure.com/courses/46578/pages/lecture-analysis-of-rc-and-lr-circuits) (<https://peralta.instructure.com/courses/46578/pages/lecture-analysis-of-rc-and-lr-circuits>), and if they still don't make sense, ask me questions.)

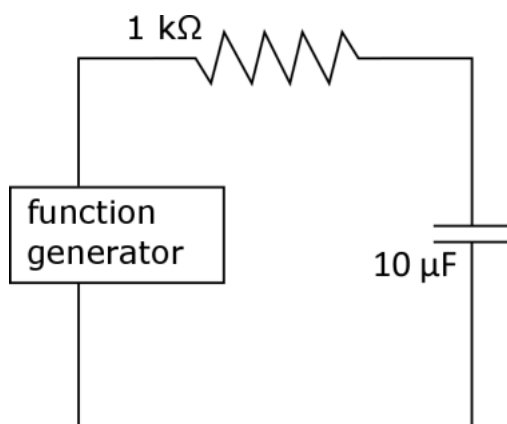




## Part A: Simple RC Circuit

In this section, you will build the following simple RC circuit and analyze it with the help of the oscilloscope and function generator.

Build the simple RC circuit below with a  $1\text{ k}\Omega$  resistor and a  $10\text{ }\mu\text{F}$  capacitor. In the next few questions, when you connect the function generator and the oscilloscope, make sure the black leads (the ground lead) are both connected to the same point, in order to avoid accidentally shorting a part of the circuit.



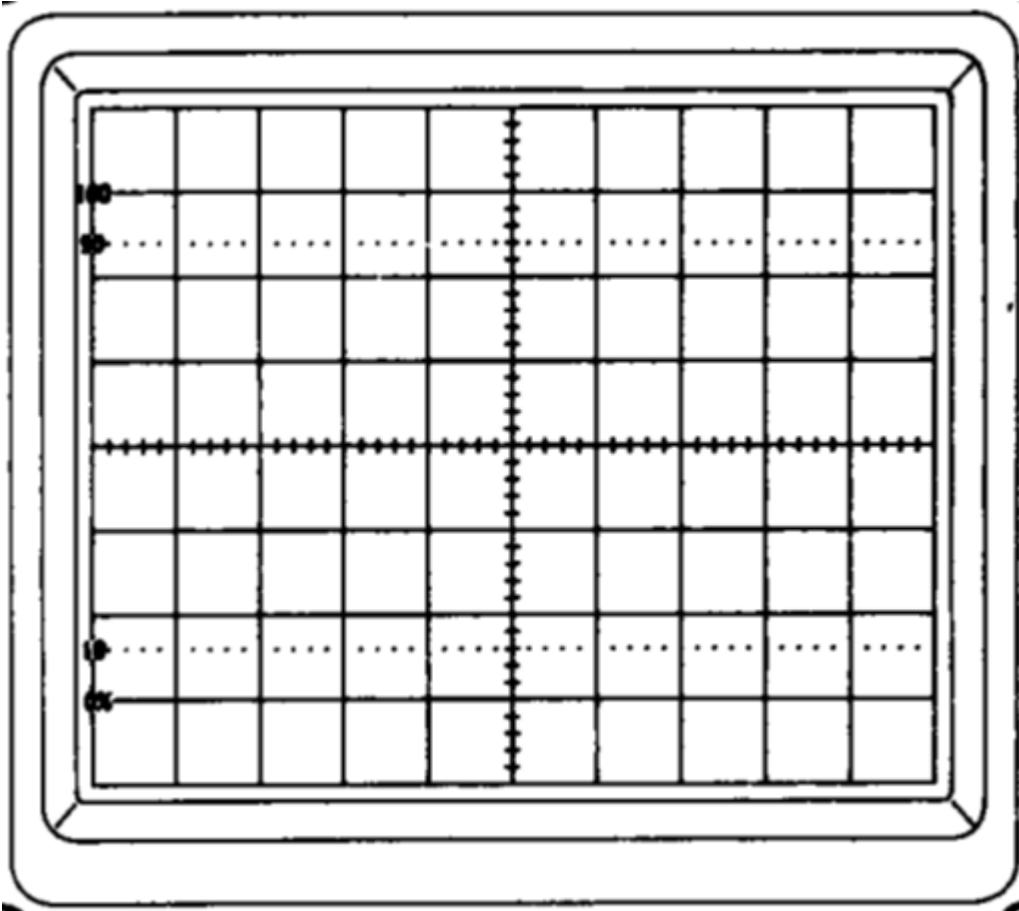
As you have seen in class, analyzing this circuit in detail involves a fair amount of math. So, let's spend some time building your intuition for this circuit.

**Q2:** The first set of intuition you need to build is for the time scale in which interesting things happen in this circuit. Work out the units of "ohm" and "farad" to show that there is *one* way to combine the resistance ***R*** and capacitance ***C*** in such a way that they give you a quantity in unit of time. This is called the **time constant** for this RC circuit. Calculate the time constant for this RC circuit (also show your work for dimensional analysis in your lab report).

You can apply a time-varying voltage on this circuit that is easier to relate to mathematically by using the square-wave function of the function generator. Apply a 10-Hz square wave (about 1 V peak-to-peak) to the circuit.

**Q3:** Measure the voltage difference across the 10  $\mu\text{F}$  capacitor (take care black leads are connected to the same point on the circuit) as a function of time (with the oscilloscope set to display at least 1 full period of the square wave). Sketch what you see on the graph below. Make sure to sketch both the input voltage (voltage measured directly from the function generator) and the voltage across the capacitor.

In the graph below, indicate the scale of the time constant you found in Q2 above. How does the time constant relate to the features you see on the plot of voltage against time?

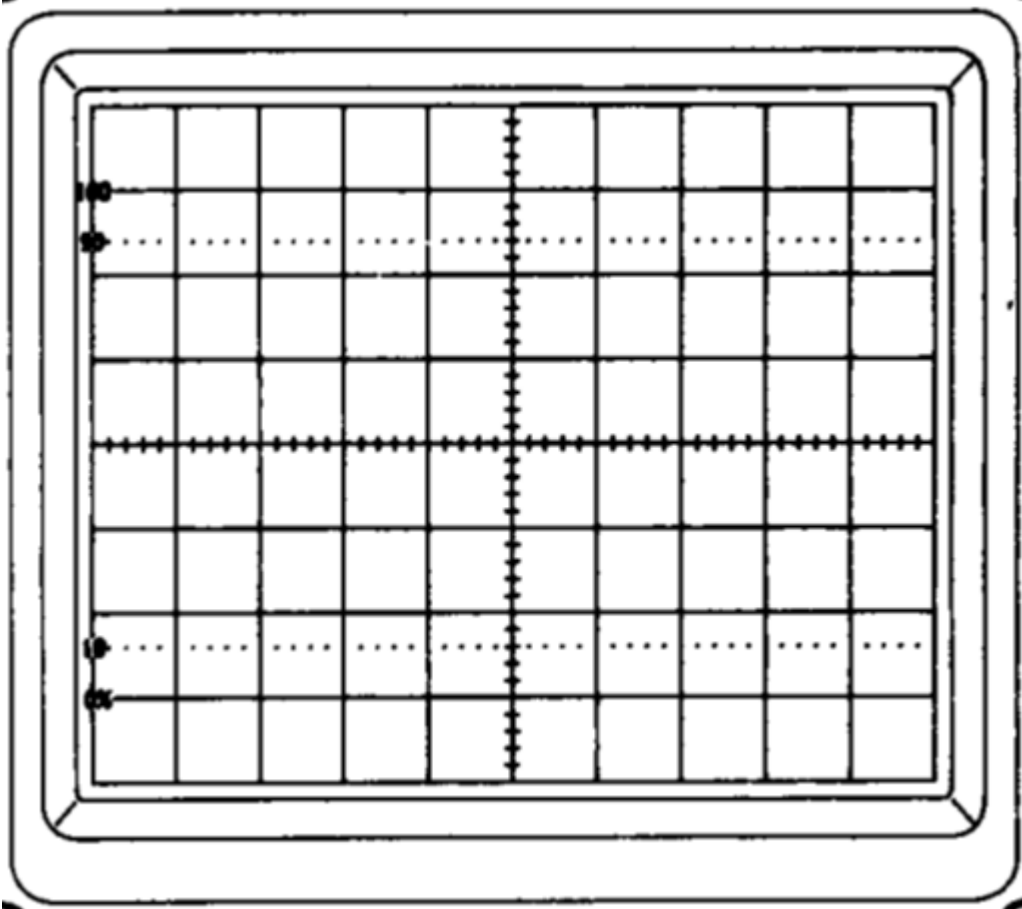


**Q4:** Which dynamic quantity in the circuit (other than the voltage across the capacitor itself) is the voltage across the capacitor most directly related to? Explain.

Suppose instead of what you measured in Q3 and Q4, you want to measure the *current* through the circuit as directly as possible.

**Q5:** Explain (in your lab report) how you would connect your instruments to measure the current through the circuit as directly as possible. Include all necessary details, and if necessary, draw (circuit) diagrams. Describe how you would connect the probe leads (the pair from the function generator, and the pair to the oscilloscope).

**Q6:** Connect your instruments as you described in Q5, and measure the current through the circuit as a function of time. With the oscilloscope set to display at least 1 full period of the square wave, line up the input voltage in this graph same as in the graph in Q3, so that you can easily compare and contrast the current through the circuit and the voltage across the capacitor. Sketch the oscilloscope picture from which you can find out the current through the circuit. When you have the sketch, **call me to obtain instructor's initials. Lab reports without instructor's initials will lose points.**



**Q7:** Explain the features you see on the graph. As you write your explanation in your lab report, make sure it addresses these questions: (1) Why does the current through the circuit decrease as the capacitor charges up? (2) What is the maximum absolute voltage you see across the resistor? Does this make sense based on how the capacitor interacts with the changing applied voltage? (3) Point at the moment in time when you get maximum current through the circuit and explain why you get maximum current at that point.

## Part B: LR Circuit

*NOTE: This lab is long. If you don't have at least 100 minutes (approximately 2 hours) remaining when you start Part B, ask me for a pre-built inductor (available from previous semesters), so that you can save some time involved in building the inductor. If you have about 100 minutes left, then follow the instructions below to build your own inductor.*

Using the provided donut-shaped ferrite core and the enamel-coated copper wires, build an inductor by wrapping the wire around the ferrite core, similar to the picture on the right. Wrap the wires tightly around the ferrite core, leaving about 5 cm at the beginning and the end for making electrical connections. Some notes:

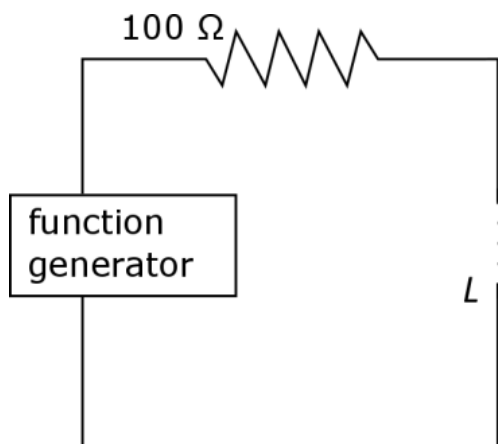


- Start with wires about 1 meter long to ensure there will be enough length of the wire.
- Wrap the wire neatly around the toroid, including as many loops as possible, going around the toroid once. The greater the number of loops, the larger the inductance will be (and larger inductance will make your measurements easier).
- When you are done wrapping, use a sandpaper to strip enamel coating from the ends of the wire, so that you can make electrical contacts for the circuit.

**Q8:** For reference, count the number of loops (depending on how your inductor was built, there should be about 50 to 100 loops) in your inductor and record it in your lab report.

In the remainder of the section, you will build the following LR circuit and analyze it with the help of the oscilloscope and function generator (keep the comparisons with the RC circuit in your mind!).

Build the LR circuit below with a  $100\ \Omega$  resistor and your inductor. In the next few questions, when you connect the function generator and the oscilloscope, make sure the black leads (the ground lead) are both connected to the same point, in order to avoid accidentally shorting a part of the circuit.



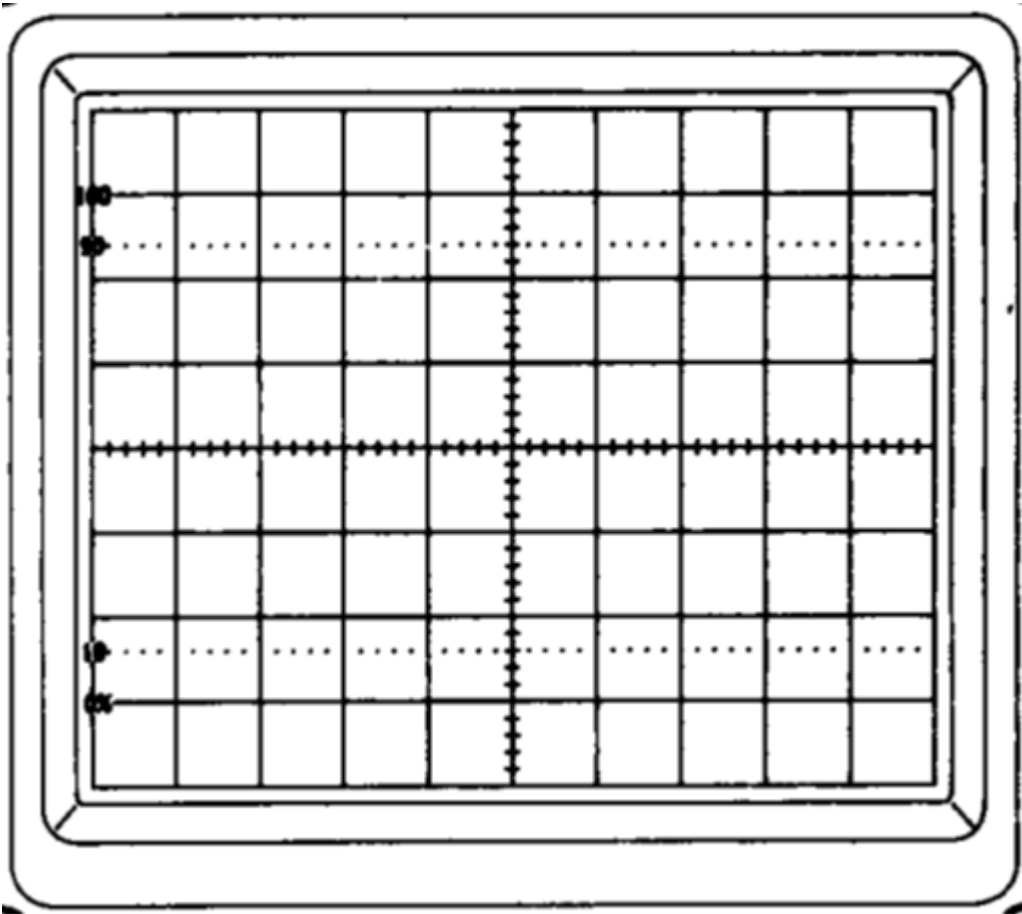
Like with RC circuit, analyzing this circuit in detail involves a fair amount of math. So, let's again spend some time building your intuition for this circuit.

**Q9:** Derive the time constant for this circuit this way. Work out the units of "ohm" and "henry" to show that there is *one* way to combine the resistance  $R$  and inductance  $L$  in such a way that they give you a quantity in unit of time. This will be the **time constant** for this LR circuit. Show your work for dimensional analysis in your lab report and write down an expression for time constant in terms of  $L$  and  $R$ .



You can apply a time-varying voltage on this circuit that is easier to work out conceptually by using the square-wave function of the function generator (similar to RC circuit). Apply a square wave (about 0.1 V peak-to-peak; you will see some issues with your voltage signal if you try to apply too large a voltage) to the circuit. Use the DC offset feature of the function generator so that in the lower half of the cycle, applied voltage is 0 volt (i.e. "off" voltage). Measuring the voltage difference across the resistor, adjust the frequency of the square wave until you see something interesting on the oscilloscope. If you simply see a square wave, you need to increase the frequency. Keep adjusting the oscilloscope time base to show several periods of the square wave on the screen.

**Q10:** Sketch what you see on the graph below. Make sure to sketch both the input voltage (voltage measured directly from the function generator) and the voltage across the resistor. *NOTE: Even though you are trying to apply a square input voltage, your actual input voltage will deviate from square shape. Sketch what you see here.*



**Q11:** Which dynamic quantity in the circuit (other than the voltage across the resistor itself) is the voltage across the resistor most directly related to? Explain.

**Q12:** With the RC circuit, you found that the time constant  $\tau = RC$  is the amount of time it takes for voltage/charge/current to change by about  $2/3$  (if you work out the circuit equations, more precisely,  $1 - 1/e$ ) of the difference in the square wave. Assuming that the time constant in this LR circuit works out the same way, measure the value of time constant using the oscilloscope picture in Q10.

Using your estimate of time constant, **calculate the inductance of your inductor**, using the formula for LR circuit time constant you obtained in Q9.

**Q13:** [OPTIONAL; skip if you don't have at least 60 minutes remaining] Explain the features you see on the graph in Q10. As you write your explanation in your lab report, make sure it addresses these questions: (1) Why does the current through the circuit not change suddenly when the applied voltage changes suddenly? (2) When the applied voltage falls to zero (i.e. square wave switches to "off" state), why does a positive current continue to flow through the resistor? [Note: For the purpose of your explanation, ignore the imperfection in the input voltage you noticed in sketching the graph in Q10.]

## Part C: RC/LR Circuit Analysis

In this section, *choose* either the RC circuit in Part A or the LR circuit in Part B to analyze it mathematically. If you have paced yourself for this lab and have about 40 minutes or remaining, you should be able to work it out, working in groups. If you are short on time, call me, so that I can give you some head start on setting up the equations.

**Q14:** For an applied voltage  $V_{in}(t)$  (as yet unknown function of time), by applying Kirchhoff's rules, find a differential equation for your circuit. If you are analyzing RC circuit, your differential equation should be in terms of  $Q(t)$  and  $dQ/dt$ . If you are analyzing LR circuit, your differential equation should be in terms of  $I(t)$  and  $dI/dt$ . Show your work in your lab report, and solve your differential equation for the highest-order derivative (that is,  $dQ/dt$  for RC circuit, and  $dI/dt$  for LR circuit).

**Q15:** Using your result in Q14, for the "discharging cycle", find a solution for your dynamic quantity ( $Q(t)$  for RC circuit;  $I(t)$  for LR circuit). That is, as an initial condition, assume that your dynamic quantity starts at some charged value ( $Q_0$  or  $I_0$ ), with the applied voltage being zero ( $V_{in}(t) = 0$  for this part). Use the method of separation of variables (call me if you are not sure how to do that), integrating both sides over definite intervals (also call me if you are not sure how to do that).

**Q16:** Again using your result in Q14, for the "charging cycle", find a solution for your dynamic quantity. That is, as an initial condition, assume that your dynamic quantity starts at zero, with the applied voltage being non-zero and constant ( $V_{in}(t) = V_0$ ). You should still use the method of separation of variables (be careful in factoring and separating correctly; terms are slightly more complicated here than Q15).

This has been a long lab; below is a pay-off for all your work above.

**Q17:** Looking at your solutions in Q15 and Q16, give some meaning to the time constant ( $\tau = RC$  for the RC circuit, and  $\tau = L/R$  for the LR circuit) you determined in Q2 and Q9 above. How would describe your time constant in terms of the role it plays in determining your dynamic quantities ( $Q(t)$  or  $I(t)$ )?

# Lab: Introduction to Driven AC Circuits

## Manual

*Note: For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on separate pieces of paper to turn in. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.*

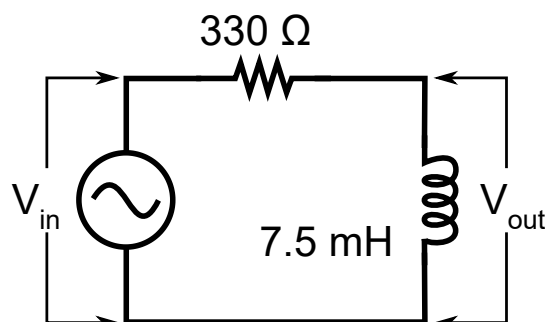
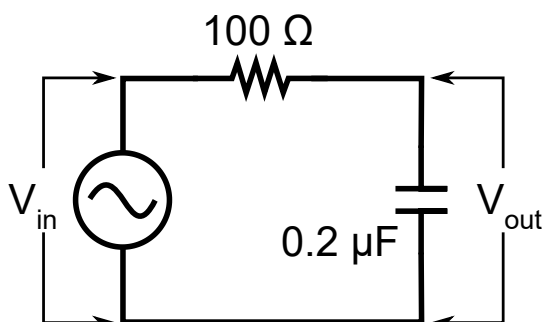
## Introduction

This is the last part of the three-part lab starting with [Lab 10: Introduction to Oscilloscope Manual](https://peralta.instructure.com/courses/46578/pages/lab-10-introduction-to-oscilloscope-manual) (<https://peralta.instructure.com/courses/46578/pages/lab-10-introduction-to-oscilloscope-manual>) and continuing from [Lab 11: Time-Dependent Circuits Manual](https://peralta.instructure.com/courses/46578/pages/lab-11-time-dependent-circuits-manual) (<https://peralta.instructure.com/courses/46578/pages/lab-11-time-dependent-circuits-manual>). Although to cover driven AC circuits in full detail we would need more time than we have remaining in the semester, we will use a convenient mathematical shortcut ("complex impedances") to analyze salient features of an important group of driven AC circuits ("filters").

**Oscilloscope Intro:** We introduced the oscilloscope in a previous lab ([Lab 10: Introduction to Oscilloscope Manual](https://peralta.instructure.com/courses/46578/pages/lab-10-introduction-to-oscilloscope-manual) (<https://peralta.instructure.com/courses/46578/pages/lab-10-introduction-to-oscilloscope-manual>)). Review the lab, particularly Part B, displaying a 1-kHz signal and looking at waveforms of different shape generated by the function generator. If necessary, take the time at the beginning of this lab to show 1 kHz signal on your oscilloscope (and call me if any portion of oscilloscope operation doesn't make sense).

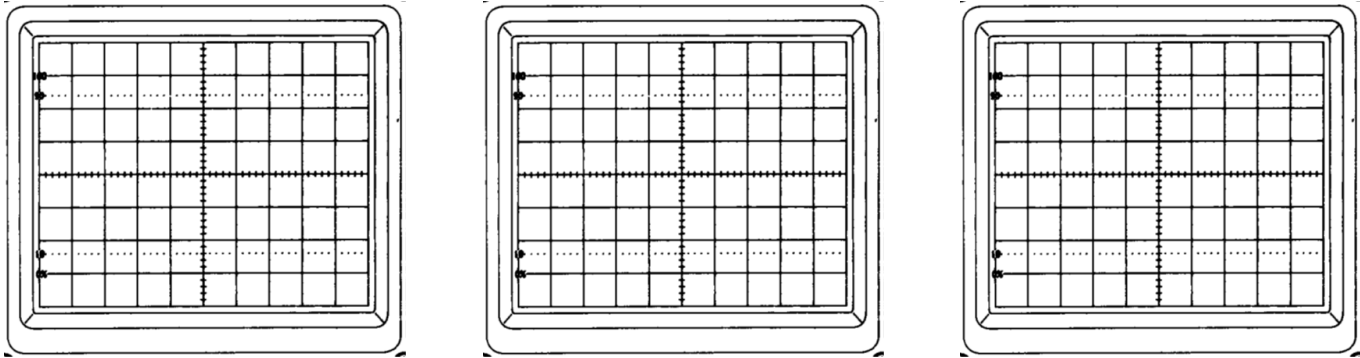
## Part A: RC/LR Filter Circuits

For this part, in the interest of time, choose one of the following two circuits. The choice of either the RC circuit or the LR circuit is up to you. The algebra you will need to do is slightly simpler for the LR circuit, but the physical circuit you will build behaves a little better for the RC circuit. If you choose the RC circuit, the circuit you will analyze can be described as a "low-pass filter". If you choose the LR circuit, the circuit you will analyze can be described as a "high-pass filter".



You will observe and analyze the behavior of the circuit at three different frequencies, 1 kHz, 10 kHz, and 100 kHz. Use the component values given in the circuit diagram (the inductor values that are available range from 5 mH to 10 mH; measure and use the actual component value of *your* inductor). If you are building the RC circuit, start your measurement with the low frequency (1 kHz); if you are building the LR circuit, start your measurement with the high frequency (100 kHz).

**Q1: OBSERVE:** Using the function generator, apply a 1-volt peak-to-peak sinusoidal voltage to the  $V_{in}$  of the circuit you built. Use the two channels of your oscilloscope to measure the  $V_{out}$  (voltage across the capacitor or the inductor, depending on which circuit you built). Display both voltages on your oscilloscope on the same scale (same volts/div and same ground). For each of the frequencies (1 kHz, 10 kHz, and 100 kHz), sketch  $V_{in}$  and  $V_{out}$  in the figure below. Make sure to label your figures.



**Q2: MEASURE:** There are two parameters that are important in the description of  $V_{in}$  and  $V_{out}$ . First is the ratio of the amplitudes of  $V_{out}$  and  $V_{in}$  ( $r = |V_{out}/V_{in}|$ ; the absolute value does have some meaning here that you'll see later). The second is the "phase difference" between  $V_{out}$  and  $V_{in}$ . Measure these two parameters for each of the three frequencies. **Record the results in a table in your lab report.** (Note on measuring "phase difference": You can measure the horizontal displacement between two waveforms as a fraction of the whole cycle, and then multiply that by  $2\pi$ . This gives the phase difference in units of radian. For the purpose of this question, don't worry too much about if this difference is positive or negative, which amounts to a difference between "leading" and "lagging" phase difference.)

Now, we are going to use a concept called "complex impedance" to calculate theoretical predictions of above two parameters. Here's the quick introduction to complex impedance. The idea behind complex impedance is that, in driven AC circuit analysis, you can treat the capacitor and inductor the *same* way you treat a resistor, with the correct impedance (a kind of generalized resistance).

The impedance of a resistor is equal to its resistance ( $Z_R = R$ ; hopefully this is unsurprising). The impedance of an inductor is  $Z_L = i\omega L$ , where  $i$  is the imaginary number (whose importance you may see in the optional exercise at the end),  $\omega$  is the angular frequency of driving signal (so, if you are applying  $f = 1000 \text{ Hz}$  signal,  $\omega = 2\pi \times 1000 \text{ Hz}$ ), and  $L$  is the inductance. The impedance of a capacitor is  $Z_C = \frac{1}{i\omega C}$ , where  $C$  is capacitance (everything else is the same as for inductor). If impedance is a kind of generalized resistance, does it make sense that the impedance of an inductor is higher for higher frequency and that the impedance of a capacitor is higher for lower frequency?

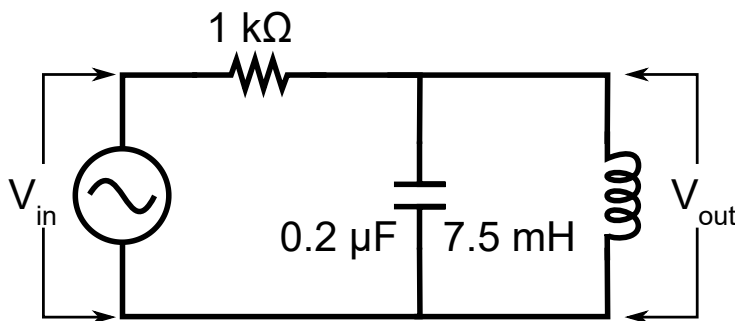
**Q3: CALCULATE:** For your circuit, using complex impedances, find an expression for  $V_{out}$  in terms of  $V_{in}$ . You may use your earlier result with voltage divider circuit. That is, take your result from [Lab 10: Introduction to Oscilloscope \(https://peralta.instructure.com/courses/46578/assignments/626337\)](https://peralta.instructure.com/courses/46578/assignments/626337) (Q11), replace the resistance of the resistor you were measuring output voltage ( $\Delta V_B$ ) from with the complex impedance of your new circuit element (inductor or capacitor). Simplify your expression, showing your work and the final result in your lab report.

**Q4:** Using your simplified expression for  $V_{out}$ , find  $r$  (as defined in Q2) by taking absolute value of both sides. Recall that, for a complex number  $z$ , the absolute value is defined as  $|z| \equiv \sqrt{z^* z}$ , where  $z^*$  is the complex conjugate of  $z$  (you obtain  $z^*$  from  $z$  by replacing every instance of the imaginary number  $i$  with  $-i$ ). You can find  $r$  as the coefficient in front of  $V_{in}$  in the expression  $|V_{out}| = r|V_{in}|$ .

**Q5: COMPARE:** Calculate the theoretical  $r$  using your answer to Q4 (if you want to check your result, an answer is provided at the end of this manual) for the three different driving frequencies. Compare your answer here to the values you measured in Q2 (remember that  $\omega = 2\pi f$  when plugging in numbers).

## Part B: Driven RLC Circuit

You saw in Part A that the circuit you built selectively "passes" a low-frequency signal (for RC filter) or a high-frequency signal (for LR filter) better. By combining the inductor with a capacitor, it is possible to build a "band-pass filter", which "passes" signals near a resonance frequency well while filtering out frequencies much below this resonance frequency or much above this frequency. Consider a circuit containing inductor, capacitor, and resistor as below:



Note that you are using the same capacitor/inductor from Part A, but a higher-resistance resistor in place of  $100\Omega/330\Omega$  resistor.

Now we are going to observe the "band-pass" behavior of this circuit, making sense of what we can, with the help of the complex impedances.

**Q6: CALCULATE:** Add impedances of the capacitor ( $Z_C$ ) and inductor ( $Z_L$ ) in parallel. Show that at a specific angular frequency  $\omega$ , these two impedances add in such a way that the equivalent impedance ( $\frac{1}{Z_{eq}} = \frac{1}{Z_C} + \frac{1}{Z_L}$ , in parallel) approaches infinity. For your component values, write down the frequency ( $f = \frac{\omega}{2\pi}$ ) at which this happens.

You have worked quite a bit with this voltage-divider arrangement. So, I hope you have some intuition that, the output voltage  $V_{out}$  is large when the impedance of the circuit element you are measuring  $V_{out}$  across (in this case, the capacitor and inductor in parallel) is large. According to this intuitive reasoning, what should you expect to see in  $V_{out}$  at the frequency  $f$  you determined in Q6?

**Q7: OBSERVE and COMPARE:** Using the function generator, apply a sinusoidal input voltage  $V_{in}$ . As you monitor  $V_{out}$ , comparing it to  $V_{in}$ , change the frequency of the signal, making sure to sweep across the frequency  $f$  you determined in Q6. (a) Record your (qualitative) observation as you change the frequency of the signal. (b) Record the frequency of the sinusoidal signal which maximizes the voltage  $V_{out}$ . How does the value of this frequency compare to the frequency  $f$  you determined in Q6?

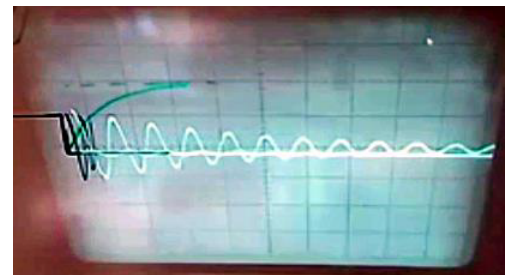
**Q8:** The resonance you observed in Q7 has a width. One way to characterize this width is to specify how far away from the resonance you need to get, in order for the output voltage to fall to half of the maximum value on resonance ("half-maximum"). Find the two frequencies at which  $V_{out}$  is half of the maximum value you observed in Q7 (which *hopefully* is close to the amplitude of  $V_{in}$ ), one above the resonance frequency and another below the resonance frequency. (For the time-being, ignore the phase shift you will see as you go off-resonance.)

At this point, call me to demonstrate and discuss your results and get my initials on your lab report. **Lab reports without instructor's initials will lose points.**

**Q9: ESTIMATE:** Without doing a complicated calculation can you estimate what  $|Z_{eq}|$  (that is, absolute value of the equivalent impedance of the capacitor and inductor in parallel) is at the two half-maximum off-resonance frequencies you found in Q8? (Hint: Ignoring the phase difference, if you replaced the capacitor and inductor with a single resistor, what resistance would it need to have, in order to produce the  $V_{out}$  you have seen?)

## Part C: "Ringing" RLC Circuit

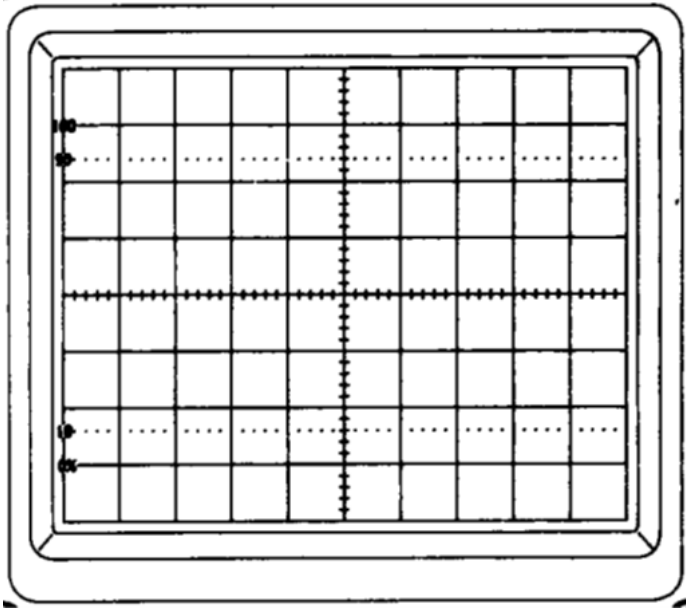
In this section, we are going to step back a little bit from the driven AC circuits and instead analyze a "ringing" RLC circuit. You are going to generate results that look like the picture on the right and see how the parameters of "ringing" change as circuit parameters are changed.



The circuit you have been working with in Part B can continue to be used for this part. You are going to use the same capacitor and inductor, but as you will be swapping out different values of resistor, have a 5-k $\Omega$  and a 10-k $\Omega$  resistor ready.

**Q10:** Drive the circuit with a square wave and watch the  $V_{out}$  "ring" (the oscillatory behavior seen in photo above). Adjust the parameters of your square wave and the oscilloscope settings until you can see the clear "ring", both the individual oscillations and the decay of the oscillation. The square wave is basically used to provide a sudden impulse. If you set up the DC offset so that the square wave is "on" for half the time and "off" for other half, you can think of it as initially charging up the capacitor and

suddenly "shorting" across the voltage supply to create a parallel-RLC circuit (but the ringing behavior can be observed for all values of DC offset, which is why it's more generally correct to think of it as a "sudden impulse"). Once you have a clear "ring", sketch your oscilloscope picture in the diagram below. Indicate the time base of your oscilloscope setting (how many seconds per division?), and measure two time-related quantities: (1) period of oscillation, and (2) how long it takes for oscillations to decrease to  $1/2$  of the initial amplitude.



Now, we are going to modify the circuit and observe how this affects the ringing behavior.

**Q11:** Swap out the 1-k $\Omega$  resistor for the 5-k $\Omega$  resistor. Adjust the square-wave and oscilloscope parameters until you can see the full, clear ringing behavior again, and once again, measure the period of oscillation and how long it takes for oscillations to decrease to half-maximum.

**Q12:** Repeat Q11, replacing the 5-k $\Omega$  resistor with a 10-k $\Omega$  resistor. Measure the period of oscillation and how long it takes for oscillations to decrease to half-maximum.

**Q13:** Compare your results in Q10, Q11, and Q12. Which parameter of "ringing" does the parallel resistor (it may not appear parallel, but if you imagine shorting across the power supply, it *is* parallel to the capacitor and inductor) affect mainly? Can you explain intuitively, based on your understanding of resistor's role in a circuit, why the circuit should "ring" for a longer period with the larger values of parallel resistor?

**Q14:** Compare your results in Q10, Q11, and Q12 with Part B. Calculate the frequency of the "ringing". How is it related with the resonance you measured with the driven RLC circuit? Explain the relationship you observe.

## Part D: Optional Analysis

In Part A, we skipped the detailed discussion of phase shift, by appealing to the expression  $|V_{out}| = r |V_{in}|$  (basically, ensure that the coefficient  $r$  will be real). It is possible to derive the phase shift algebraically by appealing to this expression instead:  $V_{out} = z V_{in}$ . Here,  $V_{out}$  and  $V_{in}$  are *complex* quantities and  $z$  is a complex coefficient. Expressing  $z$  in polar form, this expression can be written as,

$$V_{out} = (r e^{i\phi}) V_{in}.$$

The coefficient  $r$  should be the same, real value you found in Part A, and the phase angle  $\phi$  here should correspond to the phase difference you measured. You can find  $\phi$  by taking your result in Q3 (where all the quantities are still complex and you haven't taken the absolute value yet), take the complex coefficient, and use the fact that, for complex number  $z$ ,  $\phi = \arctan[\text{Im}(z)/\text{Re}(z)]$  (illustrate the complex number on a complex plane to see this is true).

At your option, calculate your theoretical  $\phi$  and see how well it compares to the experimental phase differences you measured in Q2.



# Lab: Reflection and Refraction of Light

This lab is adapted from UC Berkeley Physics 7C lab, "Reflection and Refraction of Light."

**Note:** For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on separate pieces of paper to turn in. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.

## Important Background Information About Light

Light is a wave. As is true for all waves, its **speed**  $v$ , **wavelength**  $\lambda$ , and **frequency**  $f$  are related by the **fundamental wave relation**,

$$v = \lambda f \quad \text{Eq. 1}$$

In a vacuum, the speed of light is  $3 \times 10^8$  m/s (this is the constant " $c$ "). In matter, which we usually call a **medium**, light moves more slowly than  $c$  by a factor known as the **index of refraction**  $n$  of the medium, so that,

$$v = c/n \quad \text{Eq. 2}$$

The index of refraction of air at standard temperature and pressure is  $n_{\text{air}} = 1.0003$ , meaning that in air light travels at  $1/1.0003 = 99.97\%$  of its speed in a vacuum. Water has an index of refraction of  $n_{\text{water}} = 1.33$ , so that in water light travels at  $1/1.33 = 75\%$  of its speed in a vacuum. Indices of refraction are never less than one, since in a medium with  $n < 1$  light would travel faster than  $c$ , which is impossible. In theory, indices of refraction can be infinitely high, although the highest known value of  $n$  in a medium is about 2.5 (for example, diamond has an index of refraction of 2.42).

Since the speed of light changes upon going from one medium to another, the fundamental wave relation (Eq. 1) requires that the wavelength and/or the frequency of the light also change. It turns out that *the frequency of light never changes* (except in cases of Doppler effect). (In an upper-division electrodynamics class, this would be derived from Maxwell's equations and the boundary conditions for the wave's propagation between two media.) Instead, *wavelength decreases by a factor of  $n$* . For instance, wavelengths in air are 99.97% of what they are in a vacuum (0.03% shorter), and in water they are 25% shorter. Whenever light re-enters its original medium after passing through some other medium, it returns to its original speed and wavelength.

People usually refer to a light wave by its wavelength, mostly because wavelengths are more intuitive than frequencies. What they are referring to is the **vacuum wavelength** of the light, the wavelength when  $n = 1$ . Humans can see light with a (vacuum) wavelength from about 400 nm ("nanometers",  $10^{-9}$  meters) to 700 nm, corresponding to the colors from violet to red. Pure white light contains all of

these wavelengths in equal amounts. Light with longer wavelengths is called **infrared** (700 nm to 1 mm), **microwave** (1 mm to 10 cm) or **TV/radio** (longer than 10 cm). Light with shorter wavelengths is called **ultraviolet** (400 nm to 10 nm), **X-ray** (10 nm to 10 pm, "picometers":  $10^{-12}$  meters) or **gamma ray** (shorter than 10 pm). The figure on right illustrates the electromagnetic spectrum (figure credit: [Wikimedia Commons](#)

[Commons](#) ↗

(<https://commons.wikimedia.org/wiki/File:Electromagnetic-Spectrum.png>).

In reality, the index of refraction of a medium depends a little bit on the frequency of the light in question (called "dispersion"). However, this is usually a very small effect, and we'll ignore it in this lab.

## How This Experiment Works

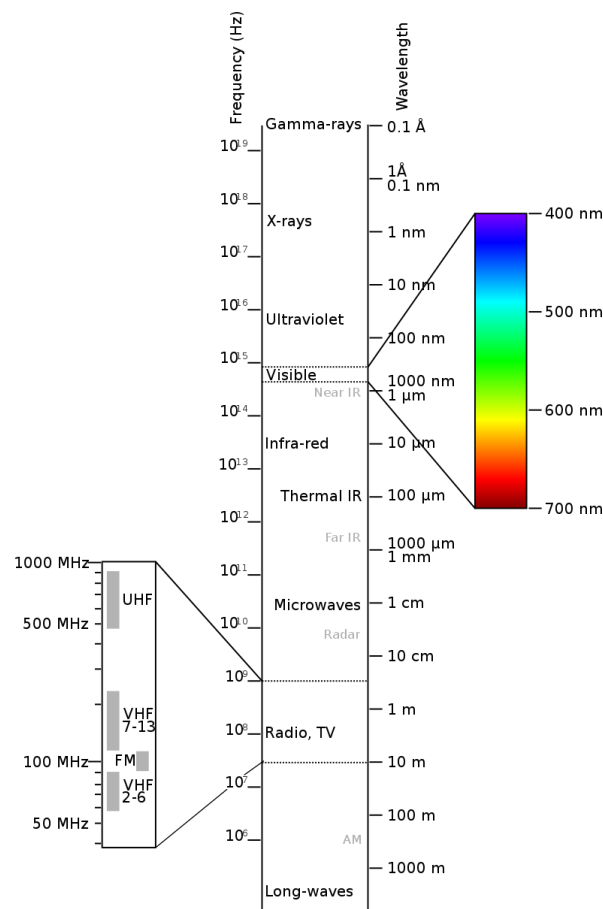
When a ray of light strikes a boundary between two media of different indices of refraction, some of this **incident** light is reflected and some is **transmitted** through the boundary (see Figure 2 below). Reflected light bounces off at the same angle as the incident light on the other side of the normal (a.k.a. surface perpendicular), as given by the **law of reflection**,

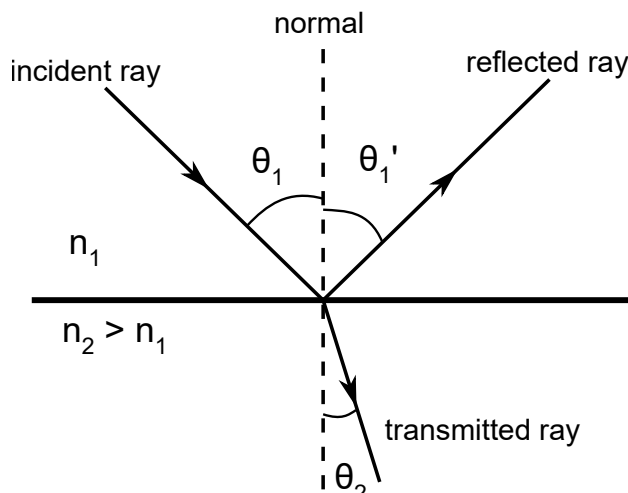
$$\theta_1 = \theta'_1 \quad \text{Eq. 3}$$

where  $\theta_1$  is the **angle of incidence** (the angle between the normal and the incident light) and  $\theta'_1$  is the **angle of reflection** (the angle between the normal and the reflected light). Transmitted light is bent, or **refracted**, upon entering the second medium, according to **Snell's Law**,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{Eq. 4}$$

where  $n_1$  is the index of refraction of the medium of the incident light,  $n_2$  is the index of refraction of the other medium, and  $\theta_2$  is the **angle of transmission** (the angle between the normal and the transmitted light, also called the **angle of refraction**).





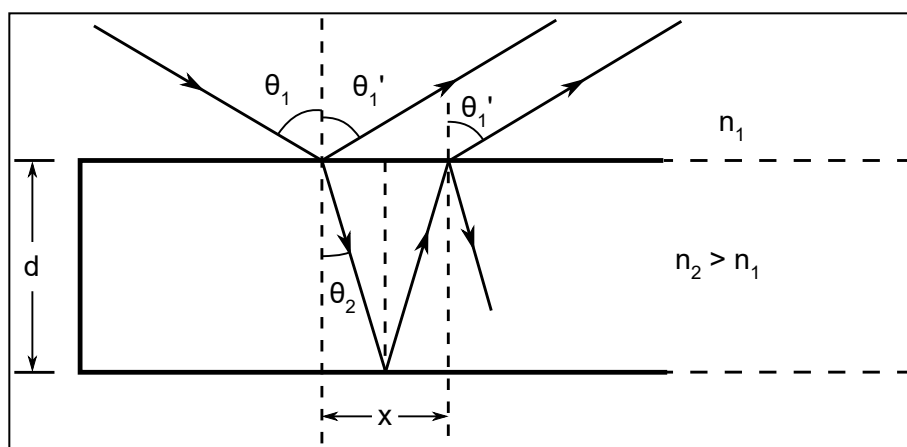
**Fig. 2.** Light paths at a surface interface.

Using these two laws and a few other properties of reflection and refraction, we are going to measure the speed of light in several materials (media) by measuring their indices of refraction. We have three methods: (1) direct refraction and (2) total internal reflection. Your instructor will tell you which materials you will be using, and for the rest of the lab manual, we will refer to these materials simply as "medium 2" (probably things like glass and plastic). "Medium 1" is air.

We are going to use a helium-neon (HeNe) laser as our light source for these experiments. Unlike the light from a light bulb or a candle, laser light doesn't diverge (spread out) as it travels away from its source. It also has only one frequency, which is  $4.736 \times 10^{14}$  Hz for the HeNe laser that we are using. (White light from bulbs and candles contain many different frequencies mixed together.) These HeNe lasers are weak and won't hurt your skin. However, they can do damage to your eyes if you look into them. (So, please don't stare into the laser.)

# Method 1: Refraction

One way to measure an unknown index of refraction of a medium is to shine light from a known medium (like air or water) into the mystery medium, and then measure its angle of incidence ( $\theta_1$ ) and angle of transmission ( $\theta_2$ ). Knowing these angles and  $n_1$ , you can then calculate  $n_2$  using Snell's law. We will do basically this with rectangular slabs of medium 2 (see Figure 3 below). The angle  $\theta_1$  is easy to measure. We can't get inside the slabs to directly measure  $\theta_2$ , but we can figure it out by measuring  $\theta_1$ , the thickness of the slab  $d$ , and the distance  $x$  (as labeled in the figure).



**Fig. 3.** Multiple reflections and refracted transmissions through slab (TOP VIEW).

Note that because these slabs have two surfaces (from air into the slab, and then back out from the slab into air) there may be lots of reflections criss-crossing the slab from side to side (but they will become fainter with each reflection and only the first few should be visible).

**Q1:** Verify your answer to [Prelab \(https://peralta.instructure.com/courses/51133/assignments/726512\)](https://peralta.instructure.com/courses/51133/assignments/726512) Question 2 (formula for  $n_{\text{slab}}$ ). Make sure you understand how to derive the expression (it's a good geometry practice).

**Q2:** Make a **prediction** before performing the procedure below: You will place the slab in the path of the laser beam, at about 15 degrees. How many beams will you see emerging on each side? Why will you see this many? (How many do you think you will see at least? Why won't you see an *infinite* number of beams?)

Turn on the HeNe laser, and place one of the slabs in the path of the laser beam at about 15 degrees (you will measure the angle precisely; it doesn't have to be exactly 15 degrees). Hold a screen up on either side of the slab to observe the emerging beams. You can measure  $x$  by placing a screen parallel to the slab and holding up a ruler between two adjacent beam spots.

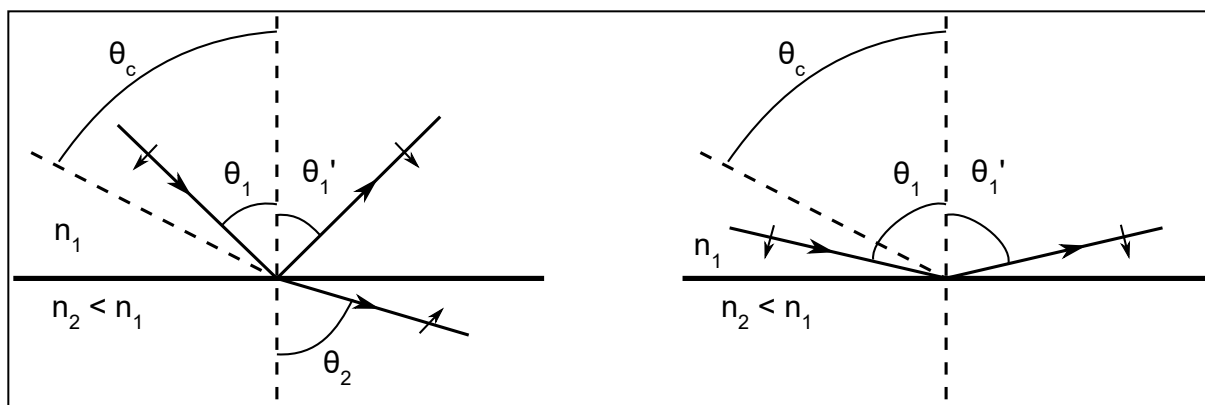
**Q3:** Measure and record the values of  $x$ ,  $\theta_1$ , and  $d$ . Describe the procedure for measuring each. [You may have to give it some thought. You have a protractor, screen, and ruler available. Pay careful attention to the orientation of the screen, and how you would use a protractor.]

**Q4:** Calculate  $n_2$  using the formula derived for Q1. Record the value of  $n_2$  in your lab report.

We will compare the values of index of refraction later, so keep this number handy.

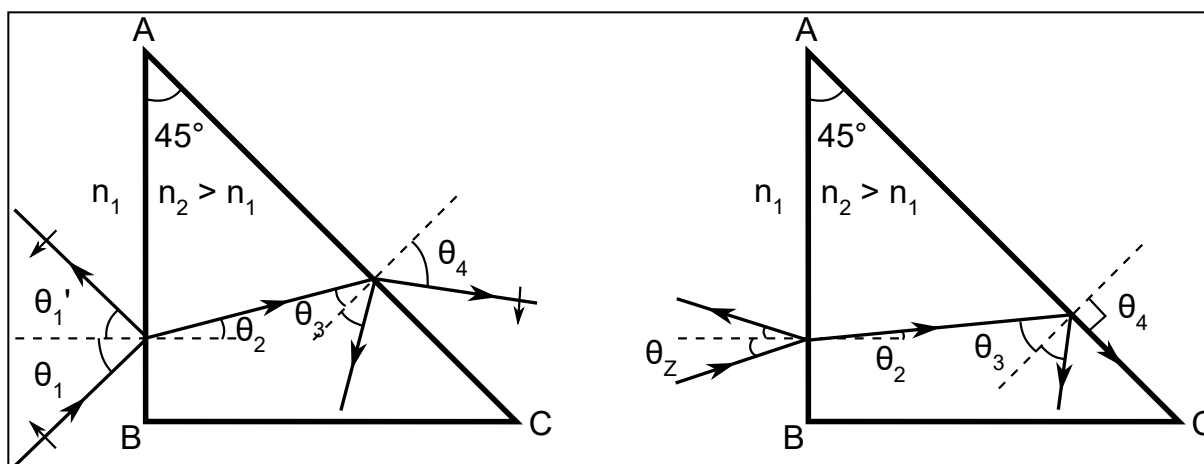
## Method 2: Total Internal Reflection

Looking at Snell's Law (Eq. 4) you can see that if  $n_1 > n_2$ , then the transmitted portion of the light is refracted away from the normal ( $\theta_2 > \theta_1$ ). At a particular angle of incidence, the light refracts a full 90 degrees from the normal, and does not enter medium 2 at all. This angle of incidence is called the **critical angle**,  $\theta_c$ , and at all values of  $\theta_1$  greater than or equal to the critical angle, all of the incident light is reflected and none is transmitted (see Figure 4). This phenomenon is called **total internal reflection**. You can solve for the critical angle from Snell's Law, by setting  $\theta_2$  equal to 90 degrees.



**Fig. 4.** Reflection and refraction with  $\theta_1 < \theta_c$  (left) and total internal reflection with  $\theta_1 > \theta_c$  (right).

We will use right-triangle prisms to find the critical angle for medium 2, from which we can solve for  $n_2$  (see Figure 5 below). If we shine light at side AB, the transmitted part will travel through the prism and emerge from side AC. By rotating the prism we can find the value of  $\theta_1$ —we'll call it  $\theta_z$ —where the transmitted beam strikes AC at exactly the critical angle. At this point no light will emerge from AC. With a little geometry we can determine  $n_2$  in terms of  $\theta_z$ .



**Fig. 5.** Double application of Snell's law in a prism (left) and total internal reflection when  $\theta_1 = \theta_z$ .

**Q5:** Verify your answer to [Prelab \(https://peralta.instructure.com/courses/51133/assignments/726512\)](https://peralta.instructure.com/courses/51133/assignments/726512) Question 5 (formula for  $n_{\text{prism}}$  in terms of  $\theta_z$ , the one angle you can measure). Make sure you understand how to derive the expression (it's a good geometry practice).

Now you are ready to do the experiment. Remove the slab from previous measurement (if it's still there).

**Q6:** You are about to put in the prism. When you do so, there will be a spot of light emerging from side AC. What other spots will you see and where? Explain using a diagram.

Replace the slab with one of the prisms. Turn it until the laser light enters on side AB, and look at the spot emerging from side AC on a screen. (If you're confused about which spot this is, cover side AC with a piece of paper and slowly draw it away, keeping the laser beam spot visible on the piece of paper as you go. This way you can trace the beam to the screen.) Look also at the spot that emerges from side BC.

**Q7:** When you rotate the prism just past  $\theta_z$ , the beam spot coming from side AC will disappear. What will happen to the intensity of the laser spot emerging from side BC? Why?

Rotate the prism slowly, and follow the spot emerging from side AC on the screen, until it just disappears. The angle of incidence to side AB is now  $\theta_z$ . Make sure that the incident beam is entering by side AB during the whole time you are rotating the prism.

For most materials,  $\theta_z$  is a very small angle, less than 10 degrees. If you are having trouble measuring it, think about the reflected part of the beam from side AB. It is reflected at the same angle as the incident beam, and is visible all the way back at the laser. With a little trigonometry, you should be able to figure out a good way of measuring the small angle  $\theta_z$  if your protractor isn't accurate enough to do it. (Try drawing a diagram!)

**Q8:** Was your protractor accurate enough to measure  $\theta_z$ ? If not, what was your method of measuring it? How does this give you an accurate measurement of such a small angle?

## Summary

Your instructor will tell you the accepted value of index of refraction for your "medium 2."

**Q9:** Which of the two methods were more accurate? What sources of error were present in this experiment?

**Q10:** How fast does light travel in your medium 2?

# Lab: Geometric Optics

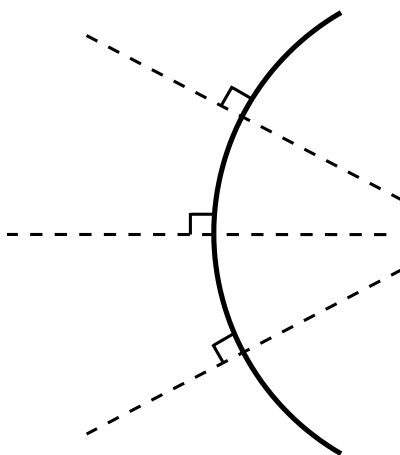
This lab is adapted from UC Berkeley Physics 7C lab, "Geometric Optics."

**Note:** For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on **separate pieces of paper to turn in**. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.

## Important Background Information About Lenses

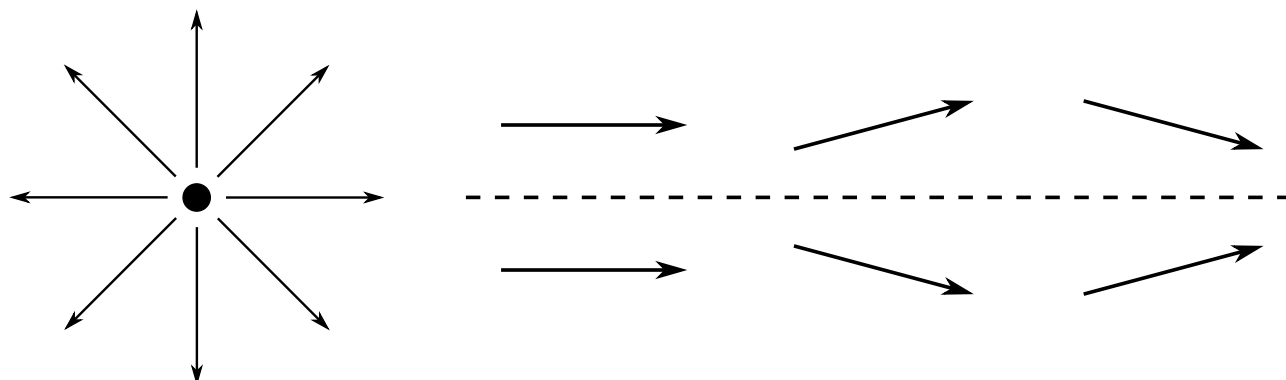
When light interacts with objects that are very large compared to its wavelength, it behaves more like a set of rays than like a wave. This ray behavior is the case when visible light interacts with lenses and mirrors, since the wavelength for visible light is 400 nm to 700 nm, whereas lab lenses and mirrors are typically several centimeters across. Approximating light as rays is called **geometric optics** (or sometimes **ray optics**), as opposed to **physical optics**, which deals exactly with the wave nature of light. We'll study physical optics in the upcoming weeks and in future labs, but for now we're going to think about light as rays, and thus do geometric optics.

Lenses and mirrors are at the heart of geometric optics. We already know about flat mirrors and "flat lenses" (flat-sided slabs of glass or some other transparent medium) from the last lab. As soon as mirrors or the sides of transparent slabs are curved, things become more complicated. Reflections are *still* at the same angle on the other side of the normal as the incident light, and transmitted light *still* bends towards/away from the normal according to Snell's law—but for curved surfaces the *normal points in a different direction at every point* (see Fig. 1 below). This means that the light striking one point on a curved mirror or lens doesn't necessarily go in the same direction as the light striking another point on the surface. This complicates things, and we need to think more carefully about light rays and exactly where they go. For the rest of this lab we will consider only lenses, but the treatment of mirrors is similar in approach.



**Figure 1:** Curved surface with different orientation of the normal at each point

In geometric optics we talk about light rays from **self-luminous objects**—objects that shine by themselves, like a candle or a light bulb. We don't worry about how these objects generate light. Light rays coming from selfluminous objects always spread out, or **diverge** (see Fig. 2a below), although rays from very distant objects (like the sun) look almost parallel by the time they reach us, and can usually be approximated as parallel. Light rays can be made to come together, or **converge**, using a lens or mirror, but no real objects will ever emit converging light (see Fig. 2b below).



**Figure 2:** (a) (on left) Light rays diverging from a self-luminous object, (b) (on right) Parallel, diverging, and converging light rays.

When studying geometric optics, we assume that all lenses are symmetric about some line called the **optic axis**. We also assume that lenses are thin, meaning that their width is small as compared to their radius of curvature. Lastly, we ignore the reflections from the surfaces of lenses. This is reasonable because most lenses are made of glass or optically similar material, and light strikes their surfaces almost normally (that is, perpendicularly). Under these conditions, only about 4% of incident intensity is reflected from a glass surface at normal incidence.

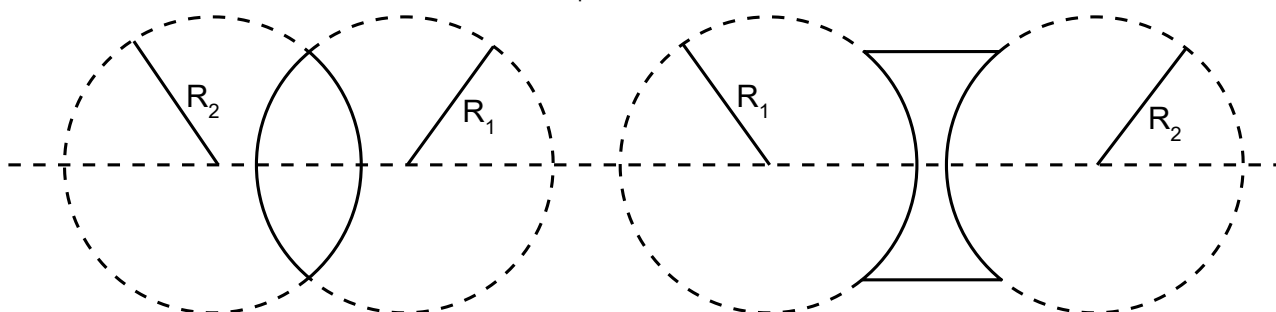
The **focal length**,  $f$ , of a lens determines how the lens affects light rays. This focal length can be calculated by the **Lensmaker's formula**.

$$\frac{1}{f} = \left( \frac{n_{\text{lens}}}{n_{\text{outside}}} - 1 \right) \times \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \quad \text{Eq. 1}$$

where  $n_{\text{lens}}$  and  $n_{\text{outside}}$  are the indices of refraction of the medium the lens is made out of (usually glass) and the medium the lens is in (usually air or water), and  $R_1$  and  $R_2$  are the radii of curvature of the front and back surfaces of the lens. (Note the [sign convention for the radius of curvature](https://openstax.org/books/university-physics-volume-3/pages/2-4-thin-lenses#fs-id1167134960329) <https://openstax.org/books/university-physics-volume-3/pages/2-4-thin-lenses#fs-id1167134960329>). If the surface is convex towards the incoming rays, the radius of curvature is positive; if the surface is concave towards the incoming rays, the radius of curvature is negative.)

[The derivation of this formula is in the textbook](https://openstax.org/books/university-physics-volume-3/pages/2-4-thin-lenses#fs-id1167134474081) <https://openstax.org/books/university-physics-volume-3/pages/2-4-thin-lenses#fs-id1167134474081>. Lenses with a positive focal length are called **converging**, and lenses with a negative focal length are called **diverging**.





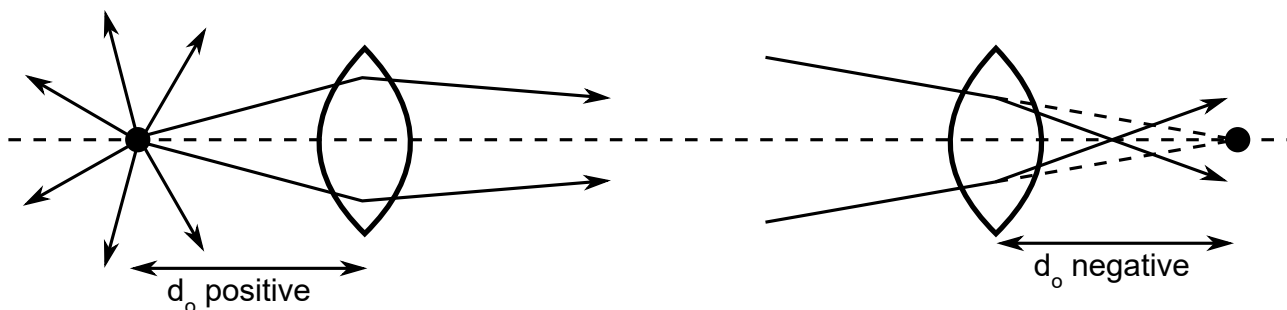
**Figure 3:** Radii of curvature for converging and diverging lenses. On left figure (converging, bi-convex lens)  $R_1$  is positive and  $R_2$  is negative. On right figure (diverging, bi-concave lens)  $R_1$  is negative and  $R_2$  is positive).

Knowing the focal length of a lens is useful because it allow us to predict where light rays passing through the lens from some object will cross and form an image. the distance from the object to the lens is known as the **object distance**,  $d_o$ , and the distance from the lens to the image is known as the **image distance**,  $d_i$ . They are related to each other and  $f$  by the **lens equation**:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \text{Eq. 2}$$

Usually the focal length of a lens will be given, and in most situations you won't have to use Eq. 1. Equation 2, however, is the relational centerpiece of geometric optics. Please memorize it.

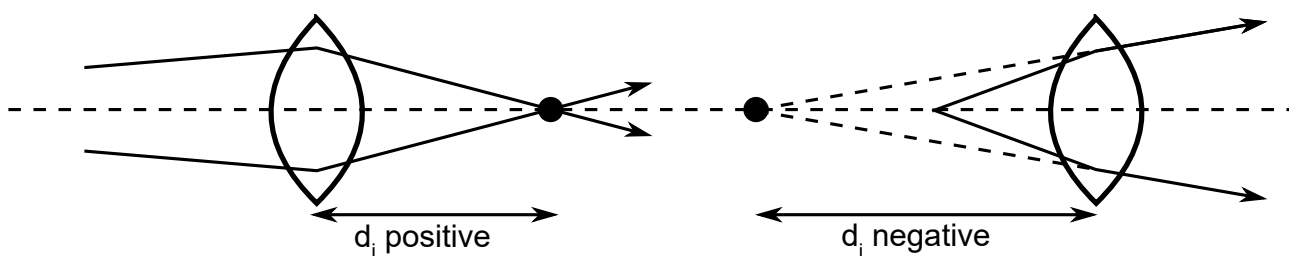
Object distances, image distances, and focal lengths can be either positive or negative. When an object is on the same side of the lens as the incoming light rays, the object distance,  $d_o$ , is positive (see Fig. 4 below). This means that the rays are diverging away from some real object towards the lens. When light rays are *converging* towards the lens, they can't have come directly from a real object, because light never converges from a real object. Instead, they probably came from some other lens that we're ignoring. We don't worry about where converging rays are coming from, and instead say that they are heading towards a "**virtual object**" located on the opposite side of the lens. The virtual object—which doesn't "really" exist—is imagined to be at the point where the rays would have crossed, if the lens hadn't been there. The object distance,  $d_o$ , in this case is negative.



**Figure 4:** Examples of real and virtual objects.

Image distance works similarly. A positive image distance means the image is on the same side as the *outgoing* rays. This is a **real image**, one at which the light rays actually converge at some point after going through the lens. You can put a screen at the point of a real image and see the focused image on the screen. A negative image distance, on the other hand, means that the outgoing light rays diverge after the lens and never cross, so there is no real image. Instead, the light rays appear to

be coming from a **virtual image** located a distance  $|d_i|$  from the lens on the opposite side to the outgoing rays. If you put a screen at the position of a virtual image, you wouldn't see anything, since the light rays don't really cross there. See Figure 5 below.



**Figure 5:** Examples of real and virtual images.

The difference between real and virtual objects/images sometimes causes confusion for students. The basic difference that you should remember is that light rays really come from or go through real objects/images, where as they don't actually come from or go through virtual objects/images.

Radii of curvature of lens (and mirror) surfaces and focal lengths have the same sign convention as image distances. If the center of curvature of a lens (or mirror) surface is on the same side as the outgoing rays, it's given a positive value. If it's on the opposite side as the outgoing rays, it's negative. If the focal length  $f$  of a lens is positive, then it has a **focal point** at a distance  $f$  away, on the same side of the lens (or mirror) as the outgoing rays. A negative focal length means that the focal point is on the opposite side to the outgoing rays.

Finally, we also define the **linear magnification**,  $m$ , of an image. This is the ratio of the linear size of the image to the linear size of the object. As derived in the textbook,

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \text{Eq. 3}$$

When  $m$  is negative, the image is formed upside down relative to the object; when it is positive, the image has the same orientation as the object.

## How This Experiment Works

Using the lens equation, we are going to study various lenses and build some simple optical devices out of them, specifically a telescope. Since we are doing geometric optics and not physical optics, we are ignoring the wave nature of light. That means we don't need a monochromatic (single-wavelength) light source, so we'll just use a regular white-light lamp instead of a laser.

This experiment comes in three parts. In Part A, you are going to measure the focal length of a converging lens using three different methods. In Part B, you will analyze a simple magnifier setup using a short-focal-length converging lens. Finally, in Part C, you are going to build a telescope using combinations of two lenses. In Parts B and C, you will estimate angular magnification, both experimentally and theoretically, and compare the two values.

## Part A: Measuring Focal Length

Find a lens marked "+100" on your table. It is supposed to be a converging lens of focal length  $f = +100$  mm. But we have to be sure of the focal lengths of our lenses before building optical devices that will work. Follow the instructions below to measure its focal length using three different methods.

### Method 1: Distant Object

This first method is a simple and practical, used often by people in a lab who pick up a converging lens and want a quick estimate of its focal length. You know that light coming from a source far away is essentially parallel, and that parallel light passing through a converging lens focuses at its focal point. A look at the lens equation says as long as  $d_o$  is much larger than  $f$ ,  $d_i$  is approximately equal to  $f$ . Since most laboratory lenses have focal lengths of millimeters or centimeters, if the object is some meters away, it can be considered "far away." So, a quick way to measure the focal length of a lens is to use the room lights as the object, hold the lens over the table, and move it up and down until the image of the lights is in focus on the tabletop. The distance between the lens and the table is then (approximately) the focal length. For a better measurement, a source further away (like the Sun) can be used instead.

**Q1:** Before doing this measurement, use the lens equation (Eq. 2) to determine the expected direction of error from the fact that instead of  $d_o$  being infinite (and  $1/d_o$  precisely zero), it is in fact finite (and  $1/d_o$  is not precisely zero). Will the measured value of  $f$  using this method be too small or too large compared to the true focal length? Show your work and result in your lab report.

**Q2:** Measure and record the value of focal length using this method. Call it  $f_1$ . Is it close to the nominal value "+100 mm"? (If not, call me, so that this can be addressed before you proceed with the lab.)

### Method 2: Measured $d_o$ and $d_i$

This second method follows on the first: by using an object (not far away) to create an image and measuring the object and image distances from the lens, you can use the lens equation to calculate the focal length. For our "object" we'll use a piece of metal with the letter "F" stamped out of it. By placing this in front of the lamp, we have a self-luminous, F-shaped object.

Using the provided optical rail, arrange the following three things in sequence: (1) self-luminous, F-shaped object, (2) converging lens with "+100" written on it, and (3) a screen (white paper mounted on cardboard). Place the lens so that the object distance  $d_o$  is between 25 cm and 40 cm (you will measure the exact value later). Move the screen behind the lens, until you find a position where a sharp, real image of F is visible on the screen.

**Q3:** Measure the object and image distances ( $d_o$  and  $d_i$ , in order) using a ruler. [Question to consider: For  $d_o$ , should you measure the distance between the lens and the light bulb, or the distance between the lens and the piece of metal with "F" stamped out?]

**Q4:** Using Eq. 2, calculate the focal length (we will call it  $f_2$ ). How does the value of this  $f_2$  compare with  $f_1$  from Q2?

Before we change this setup for Method 3, we are going to do a couple exercises that will help build up your intuition for geometric optics. Answer below questions in your lab report.

Consider swapping the distances  $d_o$  and  $d_i$  (you can do this most easily by moving the lens, rather than physically swapping the object and the screen).

**Q5: Predict before measurement:** What will happen? Will the image still appear in focus? What aspect of the image will change?

**Q6: Measure.** Do the exercise and see if you still have an in-focus image at the location of the screen. Did image change in any way that you didn't expect? Write down your answers in the lab report.

Using this setup (hopefully you have a good sized, inverted image of "F"), we will do one more exercise for something that can affect your image. Consider blocking the lens with a piece of paper immediately after the lens. You will be blocking the lens by bringing the piece of paper gradually downward (like a blind being pulled down across a window).

**Q7: Predict before measurement:** What will happen? How will the image change as you gradually block the lens with a paper, starting from the top?

**Q8: Measure.** Do the exercise and see if the image changes the way you expected. Since it's probably clear to everyone that when the paper completely blocks the lens, there is no image, focus your description on what happens in the middle of your blocking movement. For example, when the paper has covered the top half of the lens, does what you see match what you expected to see? Write down your answers in the lab report.

## Method 3: Minimum Screen Distance

The third method uses the fact that the closest an object can be to its image on the other side of the lens is  $4f$ , and at this point the object and image distances are both  $2f$  (twice the focal length).

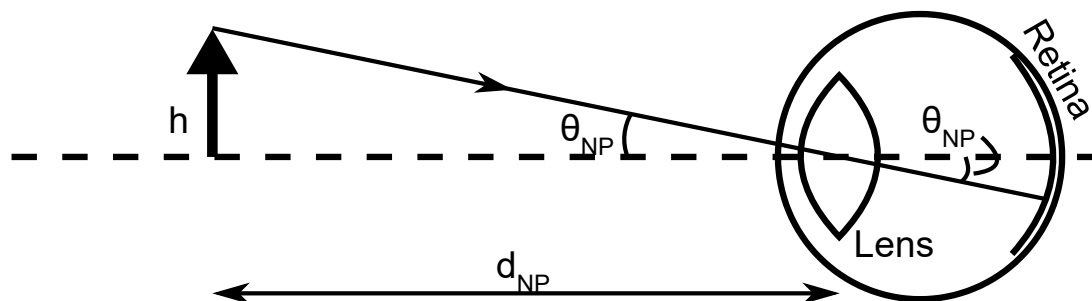
**Q9:** Review your proof for Prelab Question 5 and make sure you understand it. Record any adjustments/corrections you need to make here.

Measure this minimum distance between the object and the real image. Place the lens farther than twice the approximate focal length from the object, and place a screen to get a sharp image. As you move the lens closer to the object gradually, move the screen to follow the position of the focused image on the screen. Keep moving the lens closer to the object, up to the point where you have to move the screen farther away from the lens to get a sharp image. Adjust the lens position until the distance between the screen (with a focused image on it) and the object is at a minimum.

**Q10:** Measure this minimum distance between the screen and the object. Divide it by 4 to obtain the focal length and call it  $f_3$ . How does the value of  $f_3$  compare with  $f_2$  and  $f_1$ ? Which measurement do you believe is most accurate? Why? Write down your answers in your lab report.

## Part B: Simple Magnifier

Optical devices make images smaller or larger, so that our eyes can see them well. An eye is a combination of a lens and a built-in "screen," called a **retina**, where we sense light (see Fig. 6 below). Unlike glass lenses, the lenses in eyes have a variable focal length. When you look at something and focus on it, muscles in your eye deform the shape of your lens and change the values of  $R_1$  and  $R_2$ . As you can see from the Lensmakers' formula, this changes  $f$ . What *is* fixed in your eye is the image distance  $d_i$ —to see an object in focus, your lens must form a real image of that object on your retina, which is always the same distance from the lens.



**Figure 6:** Schematic of the eye and a light ray from an object of height  $h$ . Also illustrates the angular size,  $\theta_{NP}$ .

By putting an object close to your eye, it's easier to see fine details on it. The eye uses the angle of an image on the retina ( $\theta_{NP}$ ) to determine size, and this angle is largest for nearby objects. If you put an object too close, however, your eye muscles can't deform your lens enough to form a real image of it on the retina, and you can't focus on it. The closest point to your eye where you can still focus is called your **near point**, and is between 5 and 25 cm away from the eye for young people. It gets farther away as people age, since the lens gets stiffer and harder to deform. The angular size of an object of height  $h$  placed at the near point (a distance  $d_{NP}$ ) is,

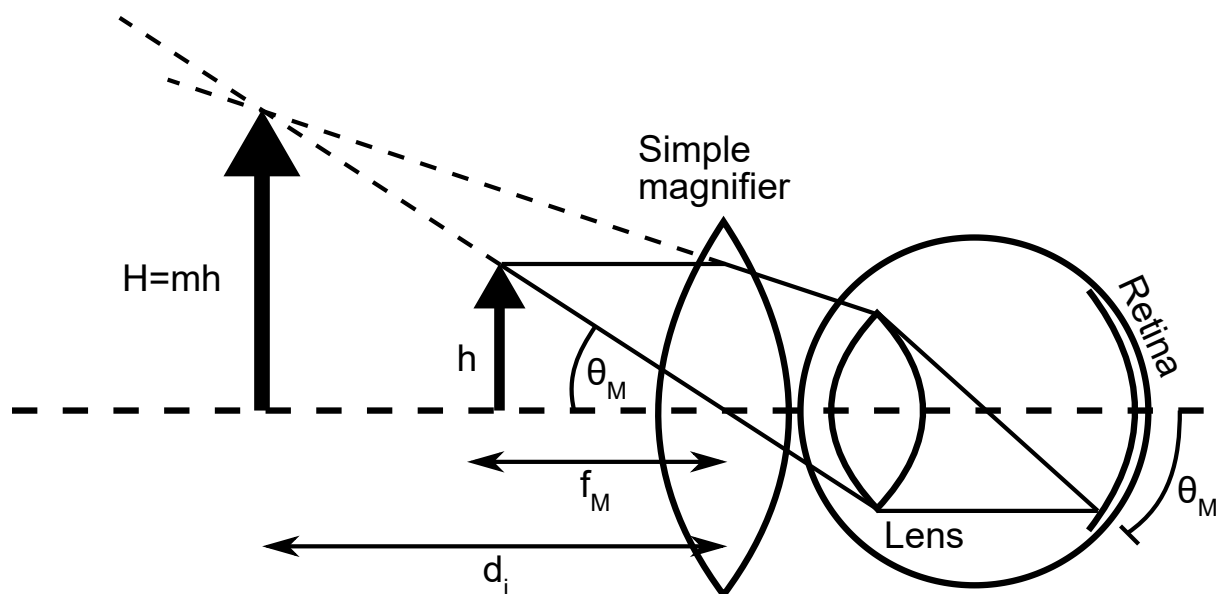
$$\theta_{NP} = \tan \theta_{NP} = \frac{h}{d_{NP}}$$

(see Fig. 6 above). We're assuming that the angle is small, but this is justified because the retina is small, so only light entering the eye at small angles is detected.

If you want to magnify an object more than is possible by placing it at your near point, the simplest way is to use an simple magnifier, a converging lens placed right in front of the eye. By putting the object slightly less than one focal length from the lens, a virtual image is formed far away and is easy to focus on. The height of this image,  $H$ , is given by the product of the height of the object and the linear magnification  $m$ . The angular size  $\theta_M$  is therefore

$$\theta_M = \tan \theta_M = \frac{mh}{|d_i|} = \frac{(-d_i/d_o)h}{-d_i} = \frac{h}{d_o} \approx \frac{h}{f_M}$$

(see Fig. 7 below). Here, we've used the absolute value of  $d_i$  because for virtual images,  $d_i < 0$ . We've used the fact that the object is essentially at the focal point of the simple magnifier. And we've also assumed that the angle made by the virtual image at the simple magnifier is the same as its angular size on the retina, treating the simple magnifier and the lens of the eye as one single combined lens, because the simple magnifier is placed so close to the eye.



**Figure 7:** The eye with a simple magnifier

The **angular magnification**,  $M_M$ , of a simple magnifier is defined as the ratio between an object's angular size with the simple magnifier,  $\theta_M$ , and its angular size at the near point without a simple magnifier,  $\theta_{NP}$ .

$$M_M = \frac{\theta_M}{\theta_{NP}} = \frac{h/f_M}{h/d_{NP}} = \frac{d_{NP}}{f_M}$$

To make an object look twice the size it appears at your near point, use a simple magnifier with a focal length that's half the distance of your near point. A converging lens with a focal length longer than your near point distance does not work effectively as a simple magnifier.

**Q11:** Start by measuring your own near point. Find your near point by holding your finger in front of one eye and slowly drawing it toward you, until you cannot focus on the finger any more. Have one of your group partners measure the distance between your eye and your finger at this position. How far away is your near point? Is it the same for both eyes?

You will be using the +50 mm lens as your simple magnifier. Quickly check that the focal length of the lens labeled "+50" is roughly 50 mm.

**Q12:** Using the  $d_{NP}$  measured in Q11, estimate the angular magnification you can obtain with your simple magnifier. Write down this value as your theoretical angular magnification using the +50 mm lens.

You can estimate an experimental value of angular magnification by following this procedure: Place a sheet of centimeter-ruled graph paper on the table. Close one eye and lean down over the paper until it is at your near point. Close your open eye, place the simple magnifier (the "+50" lens) in front of the other eye, and open the other eye. Look at a centimeter ruler through the simple magnifier and move the ruler in closer until it is in focus (about 5 cm from the lens). Now, with both eyes open, look at the superimposed image of the ruler and the graph paper. Since the linear spacing of the ruler markings and the graph paper markings are the same, you can use this to estimate the angular magnification of the simple magnifier (for example, the number of centimeter-sized squares of the graph paper that fit between one-centimeter-spaced markings on the ruler is the angular magnification of the simple magnifier. (Call me if this procedure is not clear. Quickest way to get the procedure is by having me demonstrate it for you.)

**Q13:** What is the experimental value of the angular magnification,  $M_M$ ? How does this compare with your theoretical estimate? On what can you blame the large-than-usual error?

## Part C: Telescope

There are limits on types of angular magnification possible with a simple magnifier. For example, if you want to make a very distant object appear larger, a simple magnifier doesn't help you at all (try it; all you can see is either a very blurry view of distant objects or, if you put the simple magnifier farther away, a very small real image of distant objects).

So, to clearly see very distant objects (like stars) which have a small angular size on our retina, we magnify their angular size using a telescope, an arrangement of two lenses. A telescope is composed of two lenses, a long-focal-length converging lens facing the distant object (this lens is called **objective lens**), and a shorter-focal-length converging or diverging lens facing your eye (this lens is called **eyepiece**). In a typical use of a telescope, the object is basically infinitely far away from the objective lens, so a real image is formed at almost the objective lens' focal point  $f_{OB}$ . Then, in order to form a final virtual image that is far away (so that we can look at it with a relaxed eye and it will appear focused), the eyepiece is placed a little less than  $f_{EP}$  from this image. For the angular magnification of the telescope, we cannot use the same formula as for the simple magnifier, because

for the simple magnifier, we considered the ratio of the image's angular size using the simple magnifier to its angular size at near point without any lens. With a telescope, the objects we look at are very distant ships or stars, which we cannot move to our near point. Instead, we define the angular magnification of a telescope,  $M_T$ , to be the ratio of the image's angular size using a telescope ( $\theta_T$ ) to the angular size of the object, at its current distance, without the use of a telescope ( $\theta_{OB}$ ).

**Q14:** Look up or derive the formula for angular magnification of a telescope, in terms of focal lengths of the objective lens and the eyepiece (if you are looking it up, cite your sources). What is the maximum angular magnification you can achieve with the lenses that are available at your table?

**Q15:** You can form a telescope with a converging lens as the eyepiece or a diverging lens as the eyepiece. What is the maximum angular magnification you can achieve with the lenses that are available at your table, if you use a diverging lens as the eyepiece?

**Q16:** Using the lenses available at your station, form a simple telescope. Estimate the angular magnification of your telescope (Hint: open both your eyes, so that your field of vision without telescope overlaps with your view through the telescope).

Note: On forming a simple telescope, there are two main approaches:

1. one is to carefully align two lenses on the optical rail, setting their distances carefully as you look through them (the benefit of this arrangement is, once set by someone with some skill at aligning optics, others can simply look through it)
2. the other approach is to place the eyepiece immediately over one of your eyes. Your field of view through the eyepiece will appear blurry. Hold your objective lens in the other hand and move it closer/farther from your eyepiece, until you see a clear image through the circle through the objective lens.

Try both approaches, and if you are having difficulties, call me.

## Extra: Microscope

There is actually enough equipment at your table to form a microscope (another two-lens arrangement like a telescope). If you got this far, you have the time, and you want to try it out, call me so that I can show you.



# Lab: Diffraction and Interference

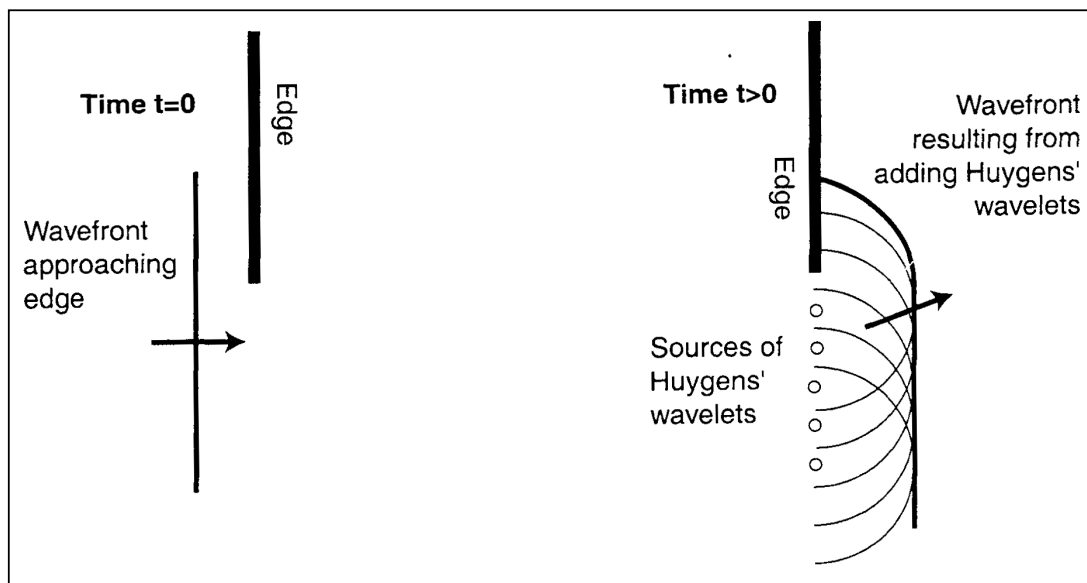
This lab is adapted from UC Berkeley Physics 7C lab, "Diffraction Gratings."

**Note:** For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on **separate pieces of paper to turn in**. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.

## Important Background Information About Interference and Diffraction

As you have learned, **interference** is the phenomenon of two waves combining their effects at the point where they intersect. What's important to know about the two waves is their **relative phase**. Waves that are **in phase** combine constructively and their effects reinforce each other. Waves that are **out of phase** combine destructively and their effects counteract each other. Only waves that have the same frequency and are coherent (maintain their relative phase for a long time) display a simple, repeated pattern of interference.

In this lab we are going to learn about a phenomenon known as **diffraction**. When a wave travels past a corner or sharp edge, it bends, or **diffracts**, around it. This happens for all waves, including light. The easiest way to understand diffraction is by using **Huygen's principle**, which deals with the cross-section of a traveling wave, known as a **wavefront**. The principle states that every point along a wavefront acts like a source of spherical wavelets (waves) with the same frequency and phase as the wavefront, and that if you draw the line or curve tangent to all those spherical wavelets after some time, you get the shape and position of the real wavefront after that same amount of time. What's important is the idea that you can treat each point on a wavefront as an independent wave source, with the same frequency and phase as all the other point sources. If you treat the points of a wavefront passing an edge as independent wave sources, and then trace the curve of tangency to those waves a little while later, you'll find that the wavefront has bent around the edge (see Figure 1 below).



**Figure 1:** A wavefront approaching an edge; the wavefront after passing the edge as deduced by the tangent line to Huygen's wavelets. Notice that the wavefront is curved after the edge, and therefore on average is moving in a different direction.

Don't confuse diffraction with **refraction**, a phenomenon we also studied in a previous lab. Refraction is also the bending of light, but it occurs when light passes from one medium into another medium with a different index of refraction. Diffraction occurs when light travels past a corner or edge in the same medium.

An interesting way to study both the interference and diffraction of light waves is by using a **diffraction grating**. This is a set of parallel slits in a piece of otherwise opaque material. The slits are much longer than they are wide. To simplify our lives, we ignore their length and consider just their width, and assume that they all have the same width and spacing. Then the only parameters that define the grating are the slit width, their spacing from each other and how many there are.

When a light wave with a flat wavefront, known as a **plane wave**, hits the grating, there are two effects on the light. First, even though each slit has some width, it acts approximately like a point wave source. This isn't Huygen's principle, it's just common sense: a narrow pinhole illuminated from the other side looks like a point source of light. Since the light from each slit is all from the same source (the plane wave) it has the same frequency and is coherent. Thus there will be a clear pattern of interference that results from these "pinhole" point sources. Second, each slit itself is really just two edges, and the plane wave light will diffract around these edges, causing what is called a **diffraction pattern**. The best way to understand the combination of these effects is to start by looking at them separately.

## Interference Effects in Diffraction Gratings

Light waves from the different slits all start out with the same phase, since they are from the same source. At a given position on the screen, the only phase difference between these waves comes from the difference in the distance they traveled to get there. If you put the screen far away from the grating compared to the distance between two slits, the angle from the normal to any screen position is approximately the same for each slit (see Figure 2 below). By geometry we then see that the difference in distance traveled for the waves from any two adjacent slits is

$$\Delta x = d \sin \theta \quad \text{Eq. 1}$$

where  $d$  is the distance between slits and  $\theta$  is the angle from the normal to the position on the screen. Assuming the grating is placed in air ( $n=1$ ) this difference in distance causes a phase difference  $\Delta\phi$ ,

$$\Delta\phi = 2\pi(\Delta x/\lambda) = 2\pi(d \sin \theta/\lambda) \quad \text{Eq. 2}$$

where  $\lambda$  is the wavelength of the light. If the phase difference at a given position is an even multiple of  $\pi$ , the interference there is constructive for all adjacent slits, since Eq. 2 holds for any two adjacent slits. That means that all the slits constructively interfere with each other, and there is a bright spot on the screen at this position. On the other hand, if the phase difference at some position is an odd multiple of  $\pi$  adjacent slits interfere destructively. Thus since each pair of slits produces zero light there, the screen will be dark at this position (even if the grating has an odd number of slits, the one unpaired slit will hardly produce any light on the screen, and we can safely ignore it). To summarize, bright spots, called interference maxima, are at screen positions  $\theta_m$  given by,

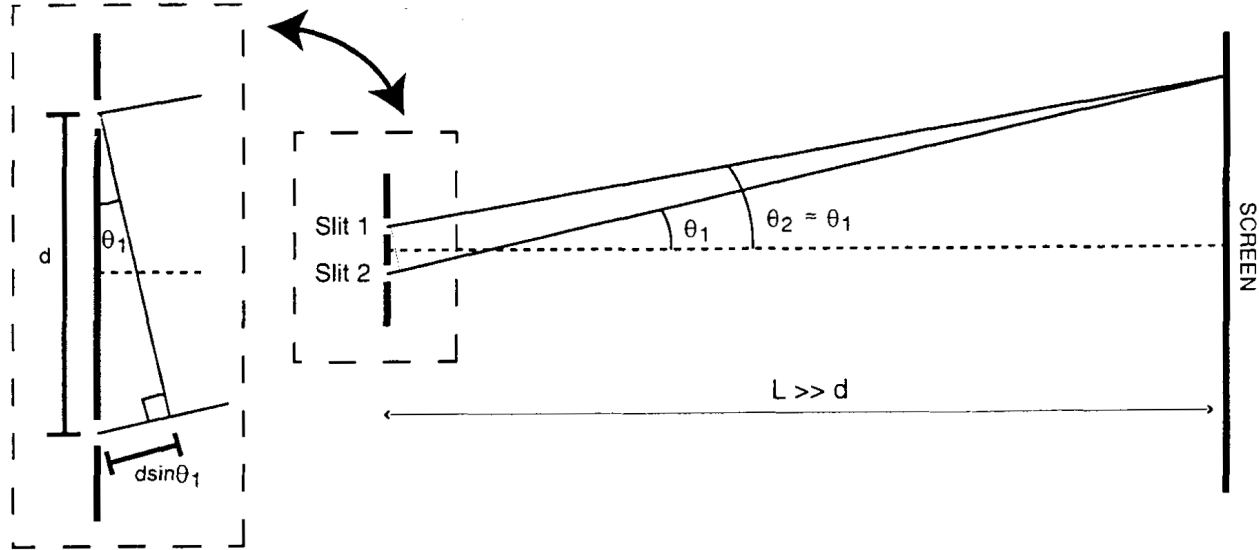
$$\begin{aligned} \Delta\phi &= \pi \times (2m) = 2\pi(d \sin \theta_m/\lambda) \\ \Rightarrow d \sin \theta_m &= m\lambda \end{aligned} \quad \text{Eq. 3}$$

where  $m$  is a positive or negative integer or zero. Dark spots, called interference minima, are at

$$\Delta\phi = \pi \times (2m + 1) = 2\pi(d \sin \theta_m / \lambda) \quad \text{Eq. 3a}$$

$$\Rightarrow d \sin \theta_m = (m + 1/2)\lambda$$

Notice that the central spot on the screen ( $\theta=0$ ) is a bright spot, corresponding to  $m=0$ .



**Figure 2:** The path difference between light from adjacent slits in a grating is  $d \sin \theta_1$ .

This calculation gives us the positions of interference minima and maxima on the screen, but not the intensity as a function of any screen position. That's a more difficult expression to derive and is done in many textbooks. For a general  $N$ -slit grating, the intensity due to interference effects is given by,

$$I(\theta) = I_0 \sin^2(N\beta) / \sin^2(\beta) \quad \text{Eq. 4}$$

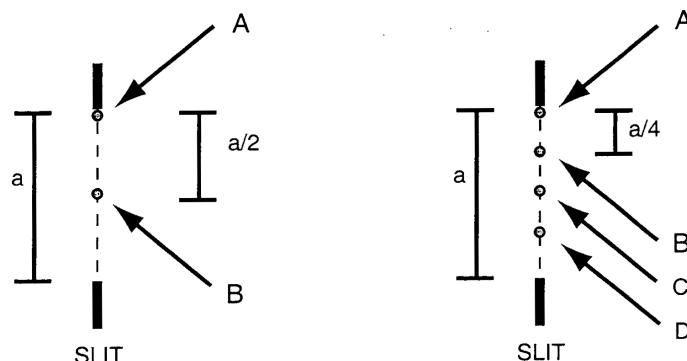
where

$$\beta \equiv (\pi d / \lambda) \sin \theta \quad \text{Eq. 4a}$$

and  $I_0$  is the intensity at the central point on the screen. A similar expression for the double-slit case was derived in lecture.

## Diffraction Effects in Diffraction Gratings

To calculate the diffraction pattern caused by the slits, we start by considering just one slit and using Huygen's principle to treat each point along its width as a source of wavelets with the frequency and phase of the incident plane wave. The net effect on the screen from this slit is the sum of the wavelets from these point sources. Because of the principle of superposition, we can add up waves in any order we choose.



**Figure 3:** Adding point sources in a diffraction slit in pairs; in quartets.

We first add together the wavelets from points **A** and **B** in Figure 3 (see above), at the top and middle of the slit. If the screen is placed far away from the slit compared to the slit width, the lines from **A** and **B** to any screen position are approximately parallel. As for interference, there is a difference in the distance traveled  $\Delta x$  for the wavelets from **A** and **B** to any screen position, and thus a phase difference  $\Delta\phi$ , given by,

$$\begin{aligned}\Delta x &= (a/2) \sin \theta \\ \Delta\phi &= 2\pi(\Delta x/\lambda) = 2\pi([a/2] \sin \theta/\lambda)\end{aligned}\quad \text{Eq. 5}$$

where  $a$  is the slit width,  $\lambda$  is the wavelength,  $\theta$  is the angle from the normal to the position on the screen and we assume the grating is in air. As you can see from Eq. 5, the wavelets from **A** and **B** will be in phase when  $\Delta\phi$  is an even multiple of  $\pi$ . But we are more interested at the moment in the out-of-phase situation. This occurs when the phase difference is an odd multiple of  $\pi$ , implying,

$$\begin{aligned}\Delta\phi &= (2m + 1)\pi = 2\pi([a/2] \sin \theta/\lambda) \\ \Rightarrow a \sin \theta &= (2m + 1)\lambda\end{aligned}\quad \text{Eq. 6}$$

where  $m$  is again a positive or negative integer or zero. The first screen position where the wavelets from **A** and **B** destructively interfere (the smallest  $\theta$ ) is found when  $m=0$ , giving us,

$$a \sin \theta = \lambda \quad \text{Eq. 7}$$

We did this calculation for points **A** and **B**, but notice that we will get the same results if we start with *any* two points in the slit that are separated by  $a/2$ . Thus when the wavelets from **A** and **B** are out of phase, so are the wavelets from all other pairs of points  $a/2$  apart. Now, here's the trick to this calculation: if we first add together the light from each  $a/2$ -separated pair of points, and then add all the results together, we will have added up the light from each point in the slit. But we've just seen that at the angle given by Eq. 7 above, each pair of points separated by  $a/2$  destructively interferes and gives zero light. By adding any number of zeros together, we still get zero. Therefore Eq. 7 tells us the position on the screen where all the light through the slit destructively interferes. This dark spot position is called a **diffraction minimum**.

We can also start by adding together quartets of points **A**, **B**, **C**, and **D** separated by  $a/4$ . In this case, the first position where wavelets from all four sources destructively interfere occurs at a  $\sin\theta=2\lambda$ . (Here we have just replaced the point separation  $a/2$  in Eq. 6 with  $a/4$ .) If we added points in groups of six, separated by  $a/6$ , we would get yet another diffraction minimum, this one at a  $\sin\theta=3\lambda$ . In general, we can find diffraction minima at

$$a \sin \theta_n = n\lambda \quad \text{Eq. 8}$$

where we are now using  $n$  as our positive or negative integer, to avoid confusion with the  $m$  of interference maxima and minima. (Don't confuse this  $n$  with the index of refraction!) Note that  $n$  cannot be zero in Eq. 8; the central point on the screen is a bright spot, as for interference effects. Diffraction maxima occur approximately at

$$a \sin \theta_n = (n + 1/2)\lambda \quad \text{Eq. 9}$$

plus one at  $\theta=0$  (the derivation of exact formula is more involved than we would like to do at the moment). Besides, as we will see in a moment, diffraction minima are usually more interesting than diffraction maxima.

As in the case of interference, this calculation doesn't tell us the intensity as a function of screen position. That calculation is also difficult and will be done in lecture (also done in your textbook). For any number of slits, the intensity at any screen position due to diffraction is given by

$$I(\theta) = I_0 \sin^2(\alpha)/\alpha^2 \quad \text{Eq. 10}$$

where

$$\alpha \equiv (\pi a/\lambda) \sin \theta \quad \text{Eq. 10a}$$

and  $I_0$  is again the intensity at the center of the screen.

Note that we have derived Eq. 8 through 10 for only one slit. However, we assume that in a many-slit grating all the individual-slit diffraction patterns overlap, since the slits are so closely spaced. Thus Eq. 8 through 10 hold for an entire grating.

A good rule of thumb to remember is that diffraction effects are only significant when the wavelength is roughly the same size as the slit. It's easy to see that diffraction is negligible for wavelengths that are much larger than the slit size. The first diffraction minimum on both sides of the central maximum occurs at

$$\theta_1 = \sin^{-1}(\lambda/a) \quad \text{Eq. 11}$$

If  $\lambda > a$  there is no solution to Eq. 11, so there is no first minimum on either side of the central maximum. That means that the whole "diffraction pattern" is just the central maximum. As  $\lambda$  becomes much larger than  $a$ , this maximum broadens out to become very flat, and the intensity on the screen becomes almost the same at all positions. Therefore, the slit just looks like an omnidirectional point source of light. If  $\lambda \ll a$ , then the first diffraction minimum is very close to  $\theta=0$ , and there is hardly any spreading of light beam (what we assume in geometric optics).

## Combining Diffraction and Interference Effects

When light passes through a grating, the two intensity patterns (Eq. 4 and 10) multiply each other. Positions where diffraction and interference both give a maximum are bright, and positions where diffraction and interference both give a minimum are dark. In addition, if one or the other effect gives a minimum, the result is also a dark spot, even if the other effect by itself would not have given a minimum: zero times anything is zero.

For instance, if you looked at some screen position and expected to see an interference maximum, but that position also happened to be where there was a diffraction minimum, then you would see a dark spot. Interference and diffraction maxima are sometimes called **orders**, and this phenomenon is known as a **missing order**. If the third interference maximum happens to be at the same place as the first diffraction minimum, making that position dark, we say that "the third interference order is missing."

The total intensity pattern (interference and diffraction effects combined) is plotted in the textbook. This pattern is called the **spectrum** of the diffraction grating. Notice that the separation of diffraction maxima is always larger than the separation of interference maxima (and for a good reason---two adjacent slits cannot be placed closer together than a single slit is wide!). Because of this difference, we refer to the broad diffraction intensity shape as the **diffraction envelope**, and call the interference maxima the **intensity peaks** (or just peaks) of the grating's spectrum. In fact, because of this spacing difference it is more common to see a missing interference order due to a diffraction minimum than the other way around, which is why diffraction minima are more interesting than diffraction maxima.

## How This Experiment Works

We're going to use our knowledge of interference and diffraction effects to measure the parameters of several gratings. Once we've determined these parameters, we'll use a high-quality grating to measure a small difference in wavelength between two different types of lasers. Measuring wavelengths in this way is called **grating spectroscopy**, and is a powerful tool that we will use again in a later lab. For this experiment we need monochromatic light with a flat wavefront, so we'll use our helium-neon (HeNe) laser ( $\lambda=633\text{nm}$ ). The fact that laser light doesn't diverge much tells us that its wavefront must be pretty flat, since a curved wavefront would either diverge or converge.

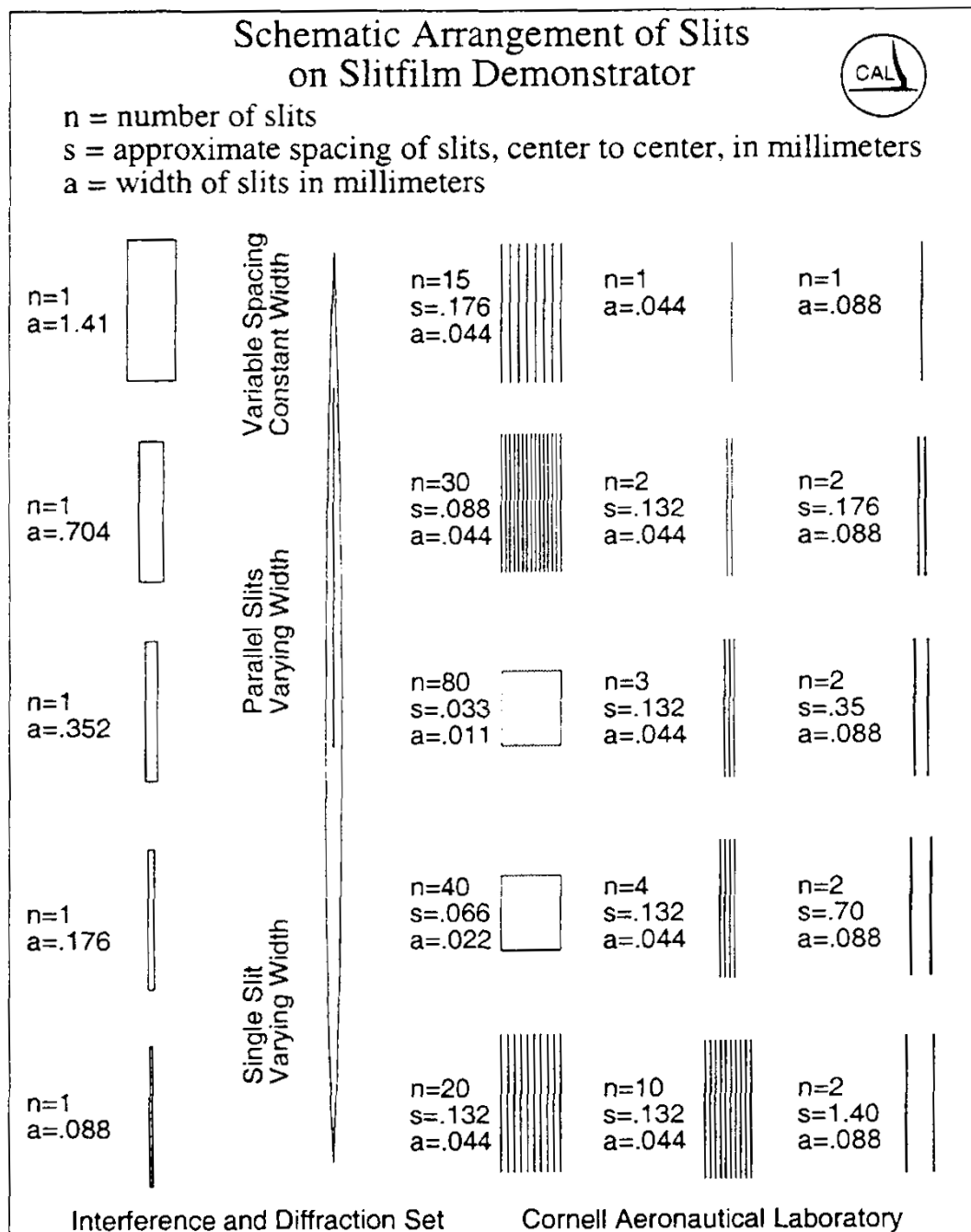
Most of our gratings are contained on a 3-inch-by-5-inch piece of glass, with different values of ***a***, ***d***, and number of slits ***N***. Figure 4 below shows a schematic drawing of the grating plate showing ***a***, ***d***, and ***N*** for each grating (*Note: take these values with a grain of salt!*). We also have a high-quality 2-inch-by-2-inch single grating with a very high value of ***N***. We'll use this for our final laser-comparison measurement. **Please don't touch the center of the high-quality grating**. The slits are delicate and will be easily destroyed by touch (and fingerprint residue you leave), so handle it only by its edges.

We'll start by measuring the angular position of intensity peaks for a single-slit grating, which has a relatively simple spectrum. From this we can deduce the slit width ***a***. Then for a multi-slit grating we'll measure peak positions and use missing orders to calculate ***d*** and ***a***. Once we understand the gratings and the spectra they produce, we'll use the high-quality grating to measure the position of the first interference peak using the diode laser light, and compare this with the position of the peak using another laser. From this we can calculate the wavelength of the second laser.

One last note: we have to be careful about how we count orders. There is always a central peak for diffraction and for interference; this is counted as order zero. Orders  $\pm 1$  are the first bright spots on either side of the central maximum. There is no zeroth-order minimum, so the first dark spots on either side of the central peak are dark orders or minima numbers  $\pm 1$ .

# Measuring Angular Positions

To measure angular positions, first measure the distance from the grating to the central peak on the screen and the distance from the central peak to the position of interest. Then use trigonometry to find  $\theta$ . For most of our gratings the small-angle approximation is valid when measuring the positions of the first few orders, but be careful about using it for orders that are far from the central peak. If the spectrum of your grating is too fuzzy to make accurate measurements, call me. I will assist you in using a converging lens to produce a focused spectrum for making a more accurate measurement.



**Figure 4:** Schematic drawing of the diffraction gratings on the plate.

# Lab Procedure

## Part A: Measuring Slit Width and Spacing

### Single-Slit

Align the HeNe laser along the optical bench, turn it on, and shine it through one of the single-slit gratings on the 3"x5" grating plate. You should see a broad grating spectrum with many bright orders.

**Q1:** Do the orders you are seeing arise from interference, diffraction, or both? Explain your answer.

Choose one of the bright spots and count which order it is. Measure its angular position on the screen and from this information deduce the slit width of your grating. Record this value as  $a_1$ . Because the orders are fuzzy, you may have some measurement error in determining their position. You can reduce this by picking a high order to measure (the high  $n$  value will help reduce error on calculated value of  $a_1$ ). If you want to try using a converging lens, call me.

**Q2:** How close is your calculated  $a_1$  to the given value? (Extra: which value should you trust more?)

**Q3: Predict before measurement:** How will the spectrum change if you use a one-slit grating with a different value of the slit width? Describe or sketch your prediction.

**Q4: Measurement:** To test your prediction, shine the HeNe laser through several other one-slit gratings and observe what happens to the spectrum on the screen. Was your prediction correct? If not, what happened?

### Double-Slit

Now shine the laser through a double-slit grating. Observe the spectrum. If you do not see a good spectrum, either choose a different double-slit grating, or call me to help you with the setup.

**Q5:** Briefly describe or sketch the double-slit grating spectrum that you observe.

**Q6: Predict before measurement:** How will the spectrum change if you use a two-slit grating with a different value of slit separation but the same slit width? Describe or sketch your prediction. Be sure to describe: (1) what will change, and (2) what won't change.

**Q7: Measurement:** To test your prediction, shine the laser through various double-slit gratings with the same slit width but different slit spacings. Observe what happens. Was your prediction correct? If not, what happened?

Shine the laser through several two-slit gratings until you see one with a clearly missing order. Record the slit separation  $d$  for this grating (see Figure 4 for reference). Now determine and record the order of the missing maximum and the order of the coincident minimum. Using the answer to Prelab Question 5, figure out the ratio of  $a$  to  $d$  for this grating.

**Q8:** Calculate the slit width for the double-slit you used in above measurement; call it  $a_2$ . You will need to use: (1) your calculated ratio above, (2) the orders of the missing maximum and the coincident minimum, and (3) the value of  $d$  you recorded above.

**Q9:** How close is your measurement of  $a_2$  to the given value? To the  $a_1$  measured in Q2? (Extra: How much do you trust the given value?)



## Part B: Measuring Wavelengths

For this part, you will need a green diode laser. Call me when you are at this part.

**Q10:** With this same two-slit grating and the values of  $d$  and  $a$  given on the grating, measure the angle of an order with the green diode laser and calculate what the wavelength of the green diode laser is. Record your answer in your lab report (show your work).

Shine laser light (either the HeNe laser or the green diode laser) through the 2"x2" high-quality grating, projecting the light beams on a screen (or a wall). Does what you see make sense? (If not, call me to have me explain it).

**Q11:** Using the high-quality grating and the first-order interference maximum you see, measure the angle of the order and calculate the wavelength of your HeNe laser and the green diode laser. Given the angle you measure, can you use small-angle approximation for this question? Why or why not?. How well do your answers here agree with your previous measurements of their wavelengths? (Note: the high-quality grating should tell you the spacing of slits ( $d$ ) in terms of "lines / length". Call me if you are having trouble figuring out the value of spacing of slits from this information.)

**Extra:** Look at various light sources (incandescent light, fluorescent light, sunlight, etc.) through the high-quality grating, by placing the high-quality grating right in front of your eye. Describe and explain what you see.



# Lab: Polarization ↕

This lab is adapted from UC Berkeley Physics 7C lab, "Polarization."

**Note:** For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on **separate pieces of paper to turn in**. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.

## Important Background Information About Polarization

Light is a transverse wave. It has the effect of creating an electric field and a magnetic field at various points in space. These fields are vectors, and are perpendicular (transverse) to the direction of propagation of the light. By convention, we define the **polarization** of a light wave as the direction of its electric field. This is just convention: we could just as easily have chosen the magnetic field for this definition, since the two fields always point at  $90^\circ$  from each other and are always in phase.

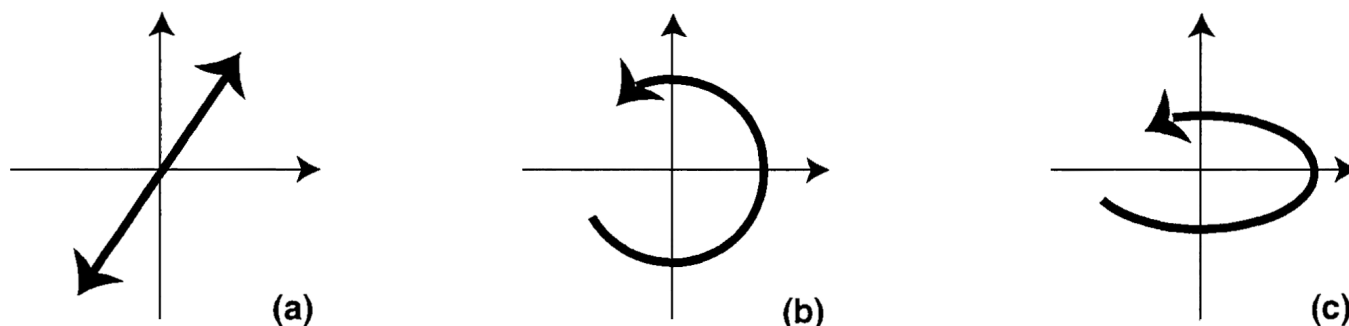
In light from most sources, the polarization (i.e. the electric field direction) changes randomly from moment to moment and from point to point. This kind of light is called **unpolarized**. In light from certain sources, however, the polarization will either stay constant in time and space or change in a predictable manner. This light is called **polarized**, and is divided into three kinds: linear, circular, and elliptical.

In **linearly polarized** light, the E-field oscillates back and forth along a fixed axis (see Figure 1a below). This is the kind of polarization we're used to seeing (in lecture). Notice that the direction of the E-field flips back and forth by  $180^\circ$  as it goes between positive and negative values. You might think that this means that linearly polarized light reverses its polarization every period. For linearly polarized light, however, we relax the definition of polarization to also include the opposite ( $180^\circ$  away) direction from the electric field. Thus linearly polarized light at  $30^\circ$  is the same as linearly polarized light at

$$30^\circ + 180^\circ = 210^\circ.$$

In **circularly polarized** light, the E-field rotates around the direction of propagation, staying a constant length (see Figure 1b below). This is still an oscillating electric field, just oscillating in direction rather than magnitude. The B-field does the same thing, turned by  $90^\circ$ . As circularly polarized light moves, the tips of the E- and B-field vectors trace out two corkscrew patterns. Circularly polarized light has two possible polarizations: **left-handed**, in which the corkscrew winds in a left-handed sense, and **right-handed**, in which the corkscrew winds in a right-handed sense.

In **elliptically polarized** light, the E-field both rotates and changes length, though it never shrinks to zero—if it did, it would be linearly polarized (see Figure 1c below). The B-field does the same thing, offset by 90°. The tips of the E- and B-field vectors trace out elliptical corkscrews (longer on one axis than the other) in space as the wave moves. Elliptically polarized light is a superposition of linearly and circularly polarized light, and both linear and circular polarizations are really just special cases of elliptical polarization.



**Figure 1:** The path traced out by the tip of the E field for linearly, circularly, and elliptically polarized light waves. The wave is going into the page. In 3D, (b) and (c) trace out corkscrew shapes as they travel.

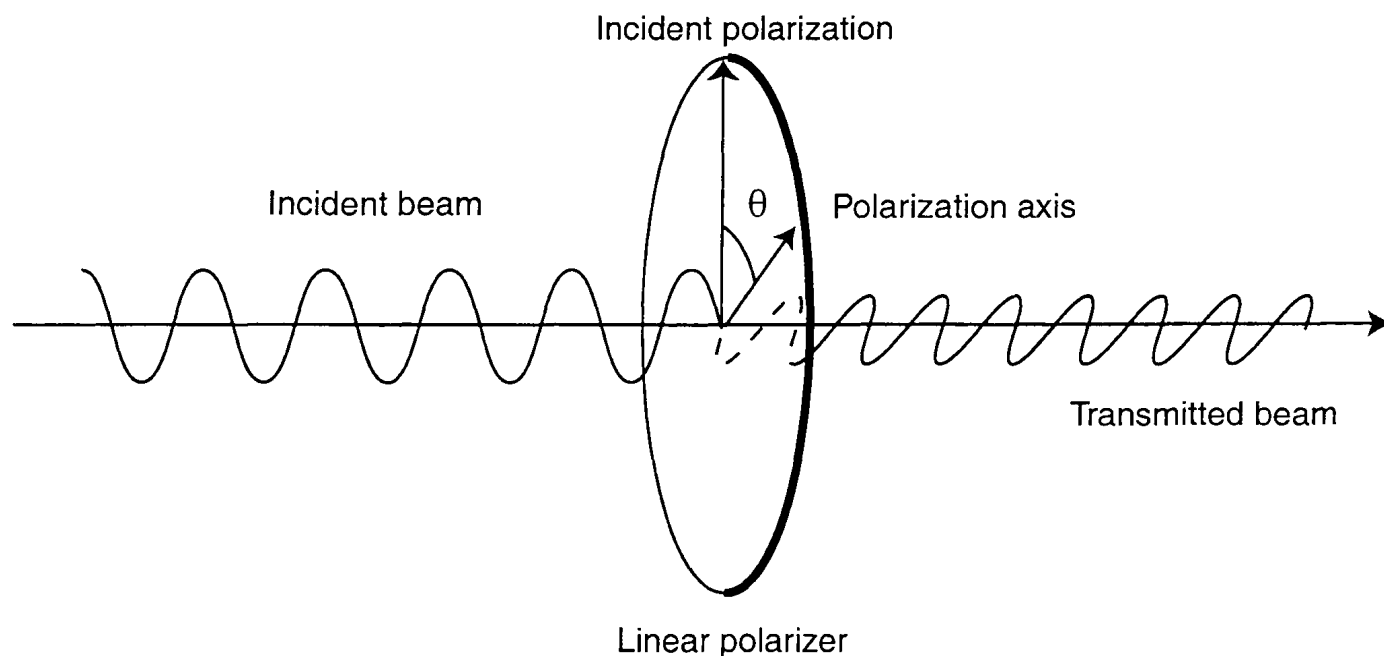
## Generating Linearly Polarized Light

The light from most normal sources (like the sun or a candle) is unpolarized, but it can be made polarized by various methods. The simplest is to shine it through a **linear polarizer**, a device that blocks all light except that with a certain linear polarization. The polarization direction that is allowed to pass is called the **polarization axis** of the polarizer. If you shine completely unpolarized light on a linear polarizer, the light that is transmitted is linearly polarized along the direction of the polarization axis.

Because linear polarizers work by blocking light, not all of the incident intensity is transmitted. We can deduce how much intensity will be transmitted by thinking of the case of an already linearly polarized light wave hitting a linear polarizer. The polarizer allows only the component of the incident electric field (i.e. polarization) along its polarization axis to pass through. The magnitude of this transmitted electric field will be the original (incident) electric field magnitude times the cosine of  $\theta$ , the angle between the polarization axis and the polarization of the incident light (see Figure 2 below). We know that the intensity of a light wave is proportional to its electric field squared, so the intensity that passes through a linear polarizer is

$$I(\theta) = I_0 \cos^2(\theta) \quad \text{Eq. 1}$$

where  $I_0$  is the incident intensity. This is the most important polarization equation, so remember it.



**Figure 2:** A linear polarizer allowing only the component of linear polarization along its polarization axis to be transmitted. The wave is traveling along the plane of the page and the polarizer is perpendicular to the page.

In a natural light source, the light emitted by each atom of the source has a specific polarization, which is a superposition of different linear polarizations. The combination of all these different linear polarizations from each atom makes the total light from a natural source unpolarized. When unpolarized light hits a linear polarizer, a different amount of each of these linearly polarized components will be transmitted. On average, however, half of the intensity of each component will get through, so the final transmitted intensity will be  $I_0/2$ . A good way to distinguish between unpolarized and linearly polarized light is to shine it through a linear polarizer. If the amount of transmitted intensity depends on the angle of the polarization axis as per Eq. 1, the light is linearly polarized. If the transmitted intensity is always half of the incident intensity for all angles of the polarization axis, it's unpolarized.

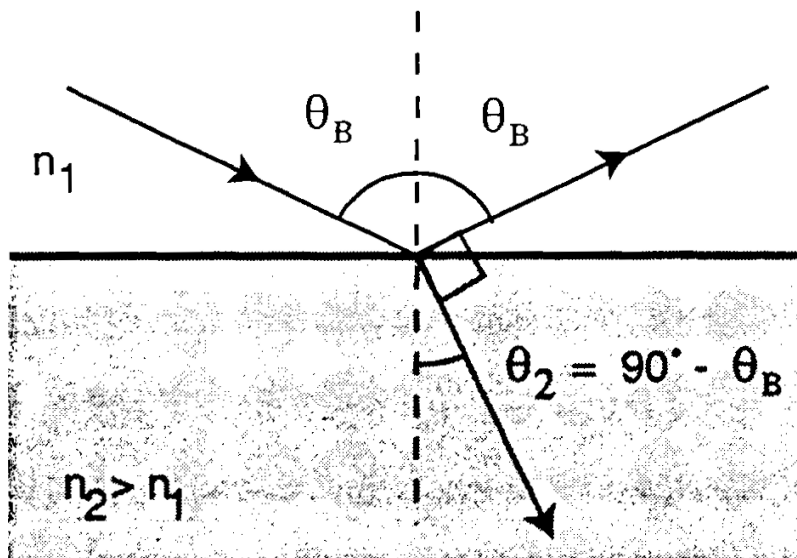
Real linear polarizers absorb a bit more light from linearly polarized light than you'd expect from Eq. 1, and more than  $I_0/2$  for unpolarized light. This "extra" absorption is because not all of the intensity along the polarization axis is transmitted, due to imperfections in the polarizer. However, even in real, imperfect linear polarizers, all incident light of any polarization (including unpolarized light) emerges linearly polarized along the polarization axis. This is the most straightforward way of generating linearly polarized light.

## Polarization using Reflection: Brewster's Angle

A second way to generate linearly polarized light is by using reflection. If you shine light of any polarization at a boundary between two media, at a particular angle of incidence the reflected beam is linearly polarized. The particular angle of incidence is called **Brewster's angle**,  $\theta_B$ . You can understand this conceptually. The reflected light from the boundary of two media is a result of

electromagnetic waves re-emitted by oscillating electrons (set in motion by the incident electric field) in the medium on the outgoing side. When a light is linearly polarized perpendicular to the **plane of incidence**, which is the plane defined by the incident and reflected beams (see Figure 3 below), the direction of oscillating electrons are set in such a way that there is no restriction on what direction the re-emitted electromagnetic waves may travel in (but interference between Huygen wavelets leads to the law of reflection). But when a light is linearly polarized in the plane of incidence, the electrons are oscillating in the plane of incidence. And a feature of [dipole radiation](https://en.wikipedia.org/wiki/Dipole#Dipole_radiation)

([https://en.wikipedia.org/wiki/Dipole#Dipole\\_radiation](https://en.wikipedia.org/wiki/Dipole#Dipole_radiation)) is (which can be shown from Maxwell's equations) that with an oscillating dipole, no electromagnetic wave is emitted along the axis of the dipole. So, when the direction of refracted light (the electrons in the outgoing medium is oscillating in the direction perpendicular to this direction) is perpendicular to the direction of reflected light, the reflected intensity becomes zero, because no electromagnetic waves are allowed to be re-emitted in the direction given by law of reflection. Brewster's angle  $\theta_B$  is the angle of incidence at which this condition is met.



**Figure 3:** A light beam striking a boundary at Brewster's angle. The plane of the page is called the plane of incidence. The reflected and transmitted beams are at right angles. The reflected beam is linearly polarized perpendicular to the plane of incidence.

Starting with Snell's law and using the fact explained conceptually above, we can derive an expression for  $\theta_B$ :

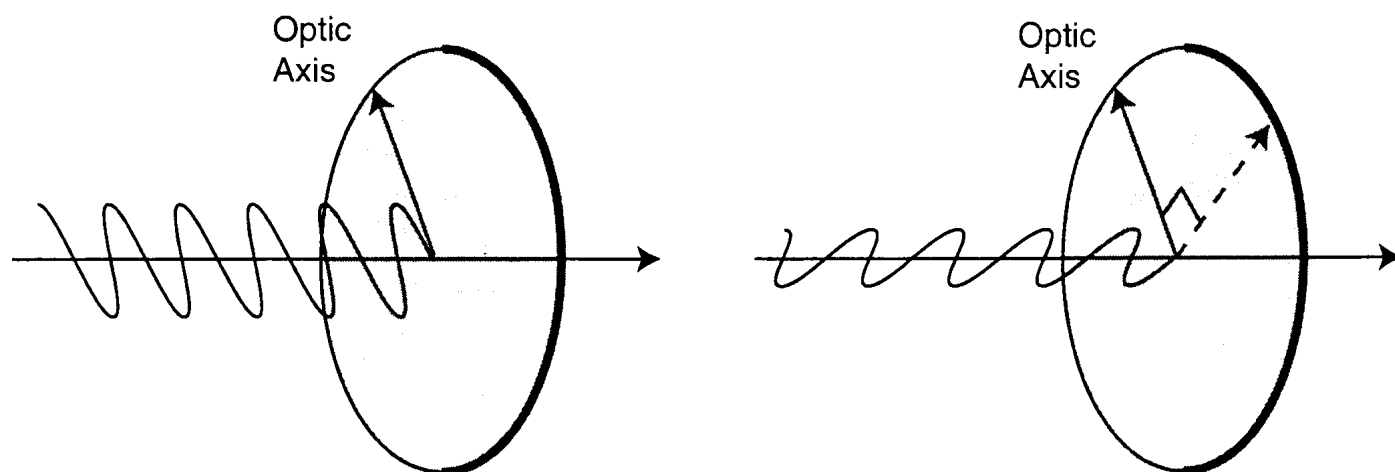
$$\begin{aligned} \theta_2 = 90^\circ - \theta_B &\implies n_1 \sin \theta_B = n_2 \sin(90^\circ - \theta_B) = n_2 \cos \theta_B \\ &\implies \tan \theta_B = n_2/n_1 \end{aligned} \quad \text{Eq. 2}$$

Brewster's angle depends on the two indices of refraction of the boundary, and so is different for each boundary. Notice, however, that Eq. 2 has a solution for any combinations of  $n_1$  and  $n_2$ , and so all boundaries have a Brewster's angle. To generate linearly polarized light from light with any other kind of polarization, we can shine the light at a boundary with an angle of incidence  $\theta_B$  for that boundary.

There's one last thing worth noting about Brewster's angle. Light that gets reflected off of rough surfaces, like desktops or car hoods (which are rough and uneven compared to the wavelength of light) is said to scatter. Scattered light is usually partially linearly polarized, since if the incident light strikes at an angle anywhere near Brewster's angle, one component of the polarization will be much more strongly reflected than the other. It won't be perfect linear polarization as it is when the angle of incidence is exactly Brewster's angle, but the imbalance in reflected intensities (between the incident light polarized perpendicular to the plane of incidence and the incident light polarized in the plane of incidence) makes the reflected beam mostly one direction of linear polarization.

## Generating Circularly and Elliptically Polarized Light

Circularly and elliptically polarized light is generated by shining linearly polarized light at a **wave plate**. Wave plates are pieces of material that are **birefringent**, meaning that they have a different index of refraction for light with different linear polarizations. Light that is linearly polarized along a direction called the **optic axis** travels through the wave plate as if it had an index of refraction  $n_1$ , and light that is linearly polarized perpendicular to the optic axis travels through the wave plate as if it had an index of refraction  $n_2$  (see Figure 4 below). A material can be birefringent intrinsically, such as crystals, whose crystalline axis determines the optic axis. An isotropic material can become birefringent when it is placed under a directional stress.



**Figure 4:** Linearly polarized light waves passing through a piece of birefringent material. The wave on the left is polarized along the optic axis and acts as if the material had an index of refraction  $n_1$ . The wave on the right is polarized perpendicular to the optic axis, and acts as if the medium had an index of refraction  $n_2$ .

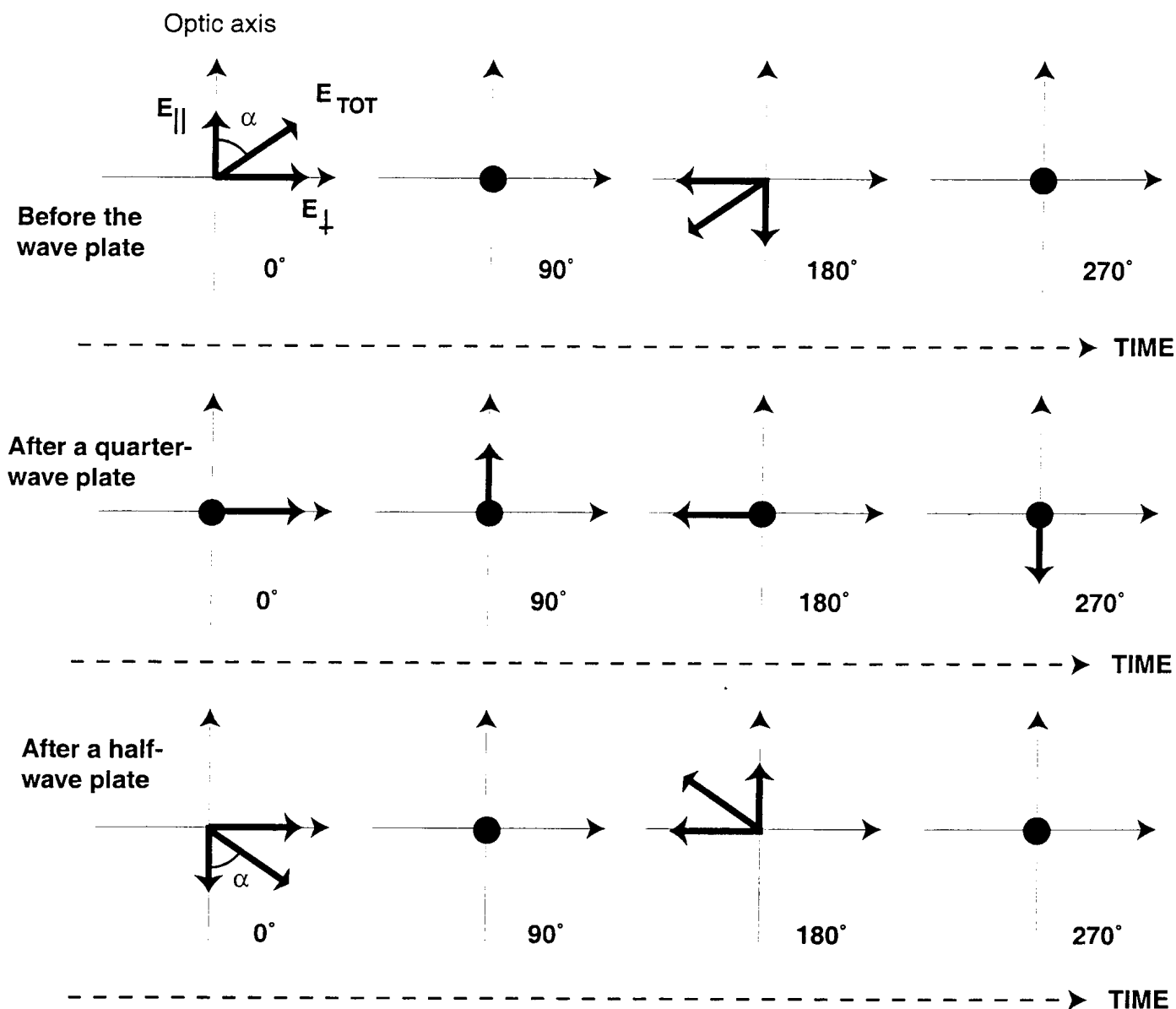
When linearly polarized light hits a wave plate, it does so with its polarization at some angle  $\alpha$  to the optic axis. We can think of this light wave as made up of two linearly polarized components, one along the optic axis and one perpendicular to it. Before hitting the wave plate, the components have the same frequency and are in phase. Once they enter the wave plate, however, there is a different index of refraction for each of them and they travel at different speeds. By the time they exit the wave

plate, they are not in phase. This phase difference between the two components is the key to generating circular and elliptical polarization, as we will now see.

The E-field (and thus the polarization) of a linearly polarized light wave is shown in Figure 5a below. The wave is travelling into the page, and the figure shows snapshots of the polarization at four points in the wave period ( $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$ ). The components along the optic axis and perpendicular to it are also shown. The two most important kinds of wave plates are called **quarter-wave** and **half-wave** plates. Their thicknesses and the ratios of their indices of refraction are chosen to change the phase between the two components by a quarter-period ( $90^\circ$ ) and a half-period ( $180^\circ$ ) respectively. In Figure 5b, the wave has travelled through a quarter-wave plate. The horizontal and vertical components of polarization (perpendicular and parallel to the optic axis, respectively) are out of phase by  $90^\circ$ . In the figure, the horizontal component has its original phase and is still maximal at  $0^\circ$  and  $180^\circ$ . The vertical component, however, is  $90^\circ$  ahead of where it originally was. Its maximum points used to also be at  $0^\circ$  and  $180^\circ$ , but now they are at  $90^\circ$  and  $270^\circ$ . If you add these two components back together with their new phase relationship, the tip of the resulting vector sweeps out an ellipse. This is elliptical polarization. If the two components are the same length—i.e.  $\alpha = 45^\circ$ —then the tip will sweep out a circle. This is circular polarization.

In Figure 5c, the wave has passed through a half-wave plate (and nothing else). In this case, the components are out of phase by  $180^\circ$ . The horizontal component again has its original phase, but the vertical component is ahead by  $180^\circ$  from where it originally was. By adding together the two components, we see that this gives us back linear polarization, but in a different direction.





**Figure 5:** TOP: The total polarization and its components for a linearly polarized wave before hitting a wave plate. MIDDLE and BOTTOM: The total polarization and its components after passing through two different kinds of wave plates. The wave is moving into the page, and the wave plate is parallel to the page. A black dot means that the E-field at that time in the wave's cycle is zero.

Linear polarizers change polarization direction by absorbing light. Wave plates, however, work by changing the relative phase between the components of polarization of a light wave. They don't absorb any light, and so they transmit 100% of incident intensity (ideally). Remember not to confuse the polarization axis of the linear polarizers with the optic axis of wave plates.

## How This Experiment Works

We are going to use linear polarizers, wave plates, and various light sources to study polarization. We'll start by looking at the effects of single and multiple linear polarizers, and then use wave plates to generate and study circular polarization. For this part of the experiment we'll use a regular incandescent light, since we don't need a collimated beam or light of a single frequency. In the

second part of the experiment we'll use our knowledge of Brewster's angle to measure the index of refraction of an acrylic slab. This will require a laser, since we'll need a well-collimated light beam.

## The Light Sensor

To measure the intensity of light transmitted through various polarizers, we'll use a light sensor. The light sensor works by measuring the voltage across a photodiode sensor as the light falls on the photodiode. We will measure all intensities in an arbitrary unit (numerical reading from the light sensor without an associated unit).

The lab room has a lot of stray light, and this creates errors in intensity measurements. To correct for stray light, you will need to do a couple of things. First, you should position the sensor as close to the last polarizer as possible to cut down on stray light reaching the sensor that hasn't passed through the polarizer. Second, you need to take a measurement of background light intensity with the sensor in position. This will give a "background intensity" measurement  $I_{BG}$  for the light sensor, and you should subtract  $I_{BG}$  from your later intensity measurements.

## Linear Polarizers and Birefringent Plates

We'll be "rotating" linear polarizers a lot, to change the direction of their polarization axis or optic axis. This means to rotate them around the axis of the beam, not the axis of their post. We'll also be measuring the percentages of "extra" intensity absorption for linear polarizers. This is the difference between the percentage of intensity that they should transmit for a given setup and the percentage that they actually do. Once you've measured this percentage, you'll have to add it back to all your measurements of transmitted intensity, after correcting for background.

## Measuring Index of Refraction Using Brewster's Angle

If you know the indices of refraction of two media, you can calculate Brewster's angle for their boundary from Eq. 2. Alternatively, if you can measure Brewster's angle and you know the index of refraction of one medium, you can calculate the index of refraction of the other medium. We're going to do this for acrylic.

At Brewster's angle, the reflected beam is entirely from the component of light polarized perpendicular to the plane of incidence. If this component is zero in the incident light, at Brewster's angle, there will be no reflected beam. We can use this fact to measure Brewster's angle. First we make the incident beam linearly polarized by shining it through a linear polarizer. The polarization axis of our linear polarizers are marked with black marks on the casing showing the orientation of this axis. We can adjust this so that the incident light is polarized in the plane of incidence. If we shine the linearly polarized light on the acrylic slab, then by turning the acrylic slab back and forth to change the angle of incidence, we can find the angle of incidence to make the reflected beam disappear. At this point we'll know that the polarization axis of the linear polarizer is in the plane of incidence, and the angle of incidence is equal to  $\theta_B$ . From this, assuming that  $n_{air} = 1$ , we can calculate  $n_{slab}$ .

# Lab Procedure

## Part A: Single Linear Polarizer

Align the incandescent light along the optical bench, turn it on, and look at the light on a screen.

**Q1: Predict before measurement:** What do you think the polarization of the lamp is? How can you tell using the linear polarizer?

**Q2: Measurement:** Insert the linear polarizer between the lamp and the screen. (You will ultimately have to have two more polarizers between the lamp and this polarizer, so leave enough room between the lamp and the screen to add them later.) Use the polarizer to determine the polarization of the lamp light. (You can do this without the light sensor.) Was your prediction correct? What did you see that lets you know (that you were correct/incorrect)?

Next, replace the screen with the sensor. Make sure that the transmitted lamp light is centered on the sensor and that the sensor is as close to the polarizer as possible. Remember that the sensor reading (in arbitrary units) is proportional to the intensity of light falling on it. You may need to change the sensor sensitivity setting; call me if you need help.

Measure the background intensity of the lab room (with the lamp off). You will need to correct your intensity measurements by this amount from here on.

Now, calculate the polarizer's percentage of "extra" absorption, by measuring the intensity of light from the lamp with and without the polarizer (remember to correct for background). You will need to add back this extra percentage to your intensity measurements from linear polarizers from here on, after you have subtracted the background intensity.

**Q3:** Record the background intensity measurement and your polarizer's percentage of extra absorption in your lab report.

## Part B: Multiple Linear Polarizers

Insert another linear polarizer in front of the previous one. (We will keep the first one, i.e. closest to lamp, in place to ensure that our light beam has linear polarization.) Rotate the second polarizer, i.e. furthest from lamp, until the transmitted intensity is at a minimum. Then, take a "background" reading with the light sensor in position. Polarizers do not absorb all frequencies of light well, so even with minimum intensity transmitted, some light will get through. You will want to subtract that out as the "background" reading (separate and different from the room light background).

Now, rotate the second polarizer until the transmitted intensity is at a maximum, and call this position  $0^\circ$ . Rotate the second polarizer again and take transmitted intensity measurements (in the arbitrary unit that your light sensor uses) every  $15^\circ$ , up to  $180^\circ$ .

**Q4:** At the position we call  $0^\circ$ , what do you know about the relationship between the polarization axes of the two polarizers?

**Q5:** Plot your measurements of transmitted intensity vs. second polarizer angle (remember to correct for the room light background and the "background" just measured). What shape should this curve have? does it have this shape?

You should find that almost no light is transmitted at  $90^\circ$  (after correcting for the "background," by definition). Rotate the second polarizer to this position and insert a third linear polarizer between the two. There should now be some transmitted intensity.

Rotate the third polarizer until the intensity is maximum and call this your new  $0^\circ$ . Rotate the third polarizer again and take intensity measurements (again in the arbitrary unit your light sensor uses) every  $15^\circ$  up to  $180^\circ$ , keeping the original two polarizers fixed.

**Q6:** Why is some intensity transmitted when you add a third polarizer?

**Q7:** Plot your measurement of transmitted intensity vs. third polarizer angle. What shape should this curve have? Does it have this shape? (Hint: look at prelab question 1.)

## Part C: Wave Plates

*[Note: We might not have necessary wave plates available for all questions in this part. In that case, certain questions will need to be skipped as announced during lab. If the questions below cannot be answered, birefringent transparency slides will be provided for producing elliptically polarized light; see Alternate Part C at the end.]*

Remove the second and third linear polarizers. Place a quarter-wave plate after the first linear polarizer. The light after the plate will have some form of elliptical polarization (which could in fact be linear or circular, since these are special cases of elliptical) depending on the angle of the optic axis. Using one more linear polarizer placed after the wave plate, you should be able to rotate the wave plate and figure out when it's at the correct angle to give circular polarization.

**Q8:** How can you use the final linear polarizer to test whether the light is circularly polarized? (Hint: think about what a linear polarizer does to circularly polarized light.) Can this test distinguish circular polarization from unpolarized light?

Using your method above, rotate the wave plate until it makes the light circularly polarized. Now put a second quarter-wave plate between the first quarter-wave plate and the final linear polarizer. You are now shining circularly polarized light from the first wave plate onto the second wave plate.

**Q9: Predict before measurement:** What do you think the polarization of the light is after the second wave plate? How can you use the final linear polarizer to check? (Hint: Look at prelab question 4.)

**Q10: Measurement:** Use the final linear polarizer to check the polarization of the light after the second wave plate. (Your setup is now one linear polarizer followed by two quarter wave plates and

then one final linear polarizer). What is the polarization now? Was your prediction in Q10 correct? Why or why not?

**Q11:** Considering that a half-wave plate is two quarter-wave plates glued together, explain why it makes sense that two quarter-wave plates would turn linear polarization back into linear polarization.

## Part D: Measuring Index of Refraction Using Brewster's Angle

Remove the incandescent light. Align the HeNe laser along the bench and shine it through a linear polarizer onto the acrylic slab. Look at the reflected beam on a screen. Rotate the linear polarizer until the axis is in the plane of incidence (the axis is shown by the black marks on the casing). Turn the acrylic slab until you can make the reflected beam disappear.

**Q12:** Measure the angle of incidence at which this occurs, and use Eq. 2 to calculate the index of refraction of the acrylic slab. Show your work.

**Q13:** What value do you measure for the index of refraction of acrylic? How close is this to the expected value of  $n_{\text{acrylic}}=1.5$ ?

## Alternate Part C: Birefringence

Do this part, if there is time remaining (or if Part C couldn't be completed due to lack of quarter-wave plates).

**Q14: Birefringence of Transparency Slides.** Using your setup measuring Brewster's angle, observe the effect of transparency slide on polarization of light. Place a transparency slide in the laser beam path between the polarizer and the acrylic slab. What do you observe on the screen (showing reflected light, if any) as you rotate the transparency slide? What can you infer from what you observe?

**Q15: Rotating Polarization with Linear Polarizers.** Suppose you have a linearly polarized light. Is it possible to rotate its polarization by  $90^\circ$  using a single linear polarizer? Is it possible using two linear polarizers? What is the maximum intensity you can achieve with two ideal linear polarizers, rotating the original linear polarization by  $90^\circ$ ?



# Lab: Speed of Light ↕

This lab is adapted from a [lab manual from Industrial Fiber Optics](https://peralta.instructure.com/courses/51133/files/5016628/download?wrap=1)

(<https://peralta.instructure.com/courses/51133/files/5016628/download?wrap=1>), maker of the electronics kit being used in the lab.

**Note:** For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on **separate pieces of paper to turn in**. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.

## Introduction - Speed of Light

Light travels at a finite speed. While this may sound intuitively true to us, who grew up in the modern era hearing about "speed of light," this fact was not widely recognized until 1700s, revealed through careful astronomical observations. Among the previously unexpected result is that it takes light about 20 minutes to cross the diameter of Earth's orbit around the Sun (read more about it on [Wikipedia](https://en.wikipedia.org/wiki/R%C3%B8mer%27s_determination_of_the_speed_of_light) ↗ ([https://en.wikipedia.org/wiki/R%C3%B8mer%27s\\_determination\\_of\\_the\\_speed\\_of\\_light](https://en.wikipedia.org/wiki/R%C3%B8mer%27s_determination_of_the_speed_of_light))).

Far more importantly for us, speed of light is the mechanism through which postulates of special relativity were discovered. In short summary: in late 1800s, James Clerk Maxwell completed the theory of electromagnetism, culminating in prediction of a wave traveling at speed  $c = 1/\sqrt{\epsilon_0\mu_0} \approx 3 \times 10^8 \text{ m/s}$ , quickly identified as light. But given that electromagnetic wave does not require a medium to travel through (you can have oscillating electric fields and magnetic fields in vacuum), this statement—constancy of speed of light—is in contradiction with how principle of relativity (laws of physics are the same in all inertial reference frames) was understood at the time. Einstein's development of theory of special relativity is the end result of reconciling this apparent contradiction, re-formulating how we should understand principle of relativity in a way consistent with the constancy of speed of light in vacuum.

Given this theoretical importance of speed of light (it really is the "universal speed limit"—that is, this is more about property of the universe we live in than property of any particular thing in this universe—light simply happens to travel at this speed limit), different techniques were developed for measuring this physical constant very precisely. Nothing recognizes this more prominently than the [1983 re-definition of meter](https://en.wikipedia.org/wiki/Metre#Speed_of_light_definition) ↗ ([https://en.wikipedia.org/wiki/Metre#Speed\\_of\\_light\\_definition](https://en.wikipedia.org/wiki/Metre#Speed_of_light_definition)), which redefined meter so that the speed of light is fixed to be exactly at  $c = 299,792,458 \text{ m/s}$  (yes, every one of those digits are significant). What this means is, experimentally, speed of light can be determined with precision of about 1 part in billion, and this precision is better than how precisely

they can measure (or maintain) the length of a [standard meter](https://en.wikipedia.org/wiki/Metre#International_prototype_metre_bar) <sup>↗</sup> ([https://en.wikipedia.org/wiki/Metre#International\\_prototype\\_metre\\_bar](https://en.wikipedia.org/wiki/Metre#International_prototype_metre_bar)).

So, all this is a good excuse for us to do a lab which demonstrates the finite propagation speed of light (we won't be measuring it with any level of precision indicated above), with the wonder of fiber optics and moderately fast electronics. Fiber optics use total internal reflection (which you learned about when we did optics), and we will review the use of oscilloscopes as necessary. For additional detailed introduction, please look at [lab manual from Industrial Fiber Optics](https://peralta.instructure.com/courses/51133/files/5016628/download?wrap=1) (<https://peralta.instructure.com/courses/51133/files/5016628/download?wrap=1>) (but I think a lot of this material is aimed at primary-school students, not college students).

## Activity Instructions

Using the **Speed of Light Apparatus** and a dual-channel oscilloscope, you can measure the speed of light in the optical fiber. The transmitter generates light pulses at a frequency of 500 kHz, transmitted through the red LED. Using an optical fiber, this light can be coupled into the detector, and the delay time of about 100 nS for the 20-meter plastic optical fiber can be detected with the dual-channel oscilloscope. This is a bit on the fast side, and the 20-MHz oscilloscope available in the lab will be barely be able to detect and measure this signal delay time.

### Equipment List

- Speed of Light Module, including: Speed of Light Apparatus, 110 VAC-to-DC power adapter, plastic fibers (15-cm and 20-meter)
- 20-MHz dual-channel oscilloscope
- 2 x oscilloscope probe

For your lab report, please write down the following on separate pieces of paper and turn in:

1. A summary of procedure (***no more than 1 page in length***) for measurement of time delay and calculation of speed of light
2. Measurement of time delay ***with uncertainty estimates***
3. Calculation of speed of light in plastic fiber and index of refraction of fiber ***with propagated uncertainty from time delay measurement***
4. One-paragraph conclusion/summary for the lab

Detailed instructions from [lab manual from Industrial Fiber Optics](https://peralta.instructure.com/courses/51133/files/5016628/download?wrap=1) (<https://peralta.instructure.com/courses/51133/files/5016628/download?wrap=1>) follows below.



## Equipment Setup

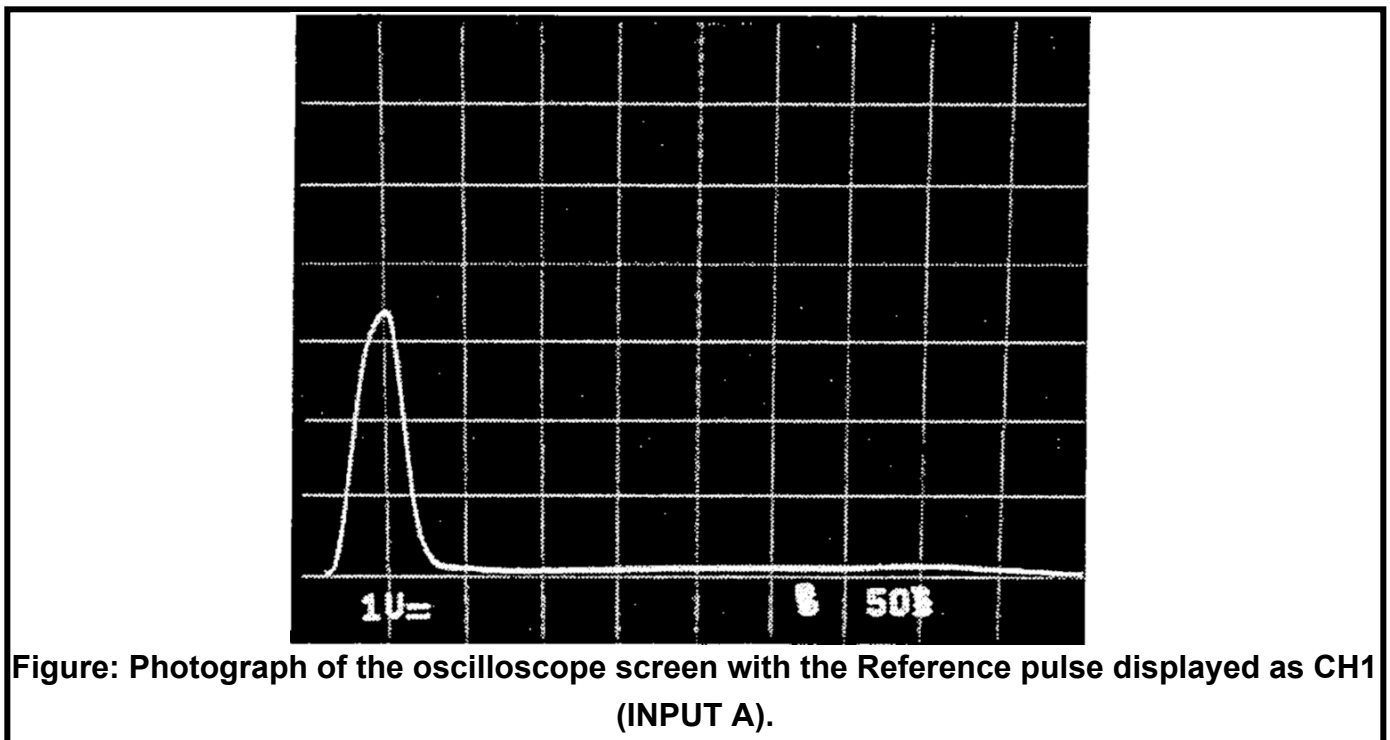
1. Turn on the oscilloscope.
2. On the oscilloscope make the following settings (for your reference, these are operating manual for our oscilloscope, [PHYS 4B - Lab - Appendix - P3502-manual-operations.pdf](https://peralta.instructure.com/courses/51133/files/5016588/download?wrap=1) (<https://peralta.instructure.com/courses/51133/files/5016588/download?wrap=1>), and Physics 4B lab manual for getting familiarized to the oscilloscope: [PHYS 4B - Lab - Introduction to Oscilloscope.pdf](https://peralta.instructure.com/courses/51133/files/5016630/download?wrap=1) (<https://peralta.instructure.com/courses/51133/files/5016630/download?wrap=1>); call me if any questions come up as you set up the oscilloscope):
  - Set TRIGGER on AUTO (not NORM).
  - Set Trigger SOURCE on INT (this makes it trigger on INPUT A)
  - Set TRIGGER SLOPE on positive slope (look at the diagram on the oscilloscope)
  - Set VOLTS/DIV on INPUT A (CH1) on 1-volt per division
  - Set VOLTS/DIV on INPUT B (CH2) on 0.5-volt per division
  - Set the input coupling of both channels on AC (the other two choices are GND and DC)
  - Set TIME/DIV on the smallest scale available (0.5 or 0.2 microsecond per division)
  - Make sure CH1 and CH2 are turned on (buttons pressed in) and ADD is not turned on
3. Connect the probe on INPUT A to the test point marked **Reference** on the Speed of Light Apparatus.
4. Connect the ground lead of INPUT A probe to the ground test point just below the **Reference** test point.
5. Connect the probe on INPUT B to the **Delay** test point on the apparatus.
6. Connect the ground lead of INPUT B probe to the ground test point just below the **Delay** test point.
7. Temporarily switch CH2 (INPUT B) input coupling from AC to GND.
8. Plug in the power for the Speed of Light Apparatus. Plug in the AC adapter into a power outlet and connect the power cable to the apparatus.
9. As soon as the AC adapter supplies power to the apparatus, the yellow LED should light up. **D3**, the fiber optic LED, should also be visible if you lean down to look sideways into the front of the blue fiber optic housing.
10. Turn the **Calibration Delay** knob on the apparatus to the 12 o'clock position.
11. Loosen the fiber optic cinch nuts on the fiber optic LED **D3** and detector **D8**.
12. Select the 15-cm length plastic fiber and insert one end of it into LED **D3** until it is seated, then *lightly* tighten the fiber optic cinch nut.
13. Insert the other end of the plastic fiber into detector **D8** until seated, then tighten its fiber optic locking nut.

Now you are ready to calibrate your apparatus. Call me if there are any questions about the above 13-step setup (so far, you haven't measured anything on your oscilloscope yet).

## Calibration

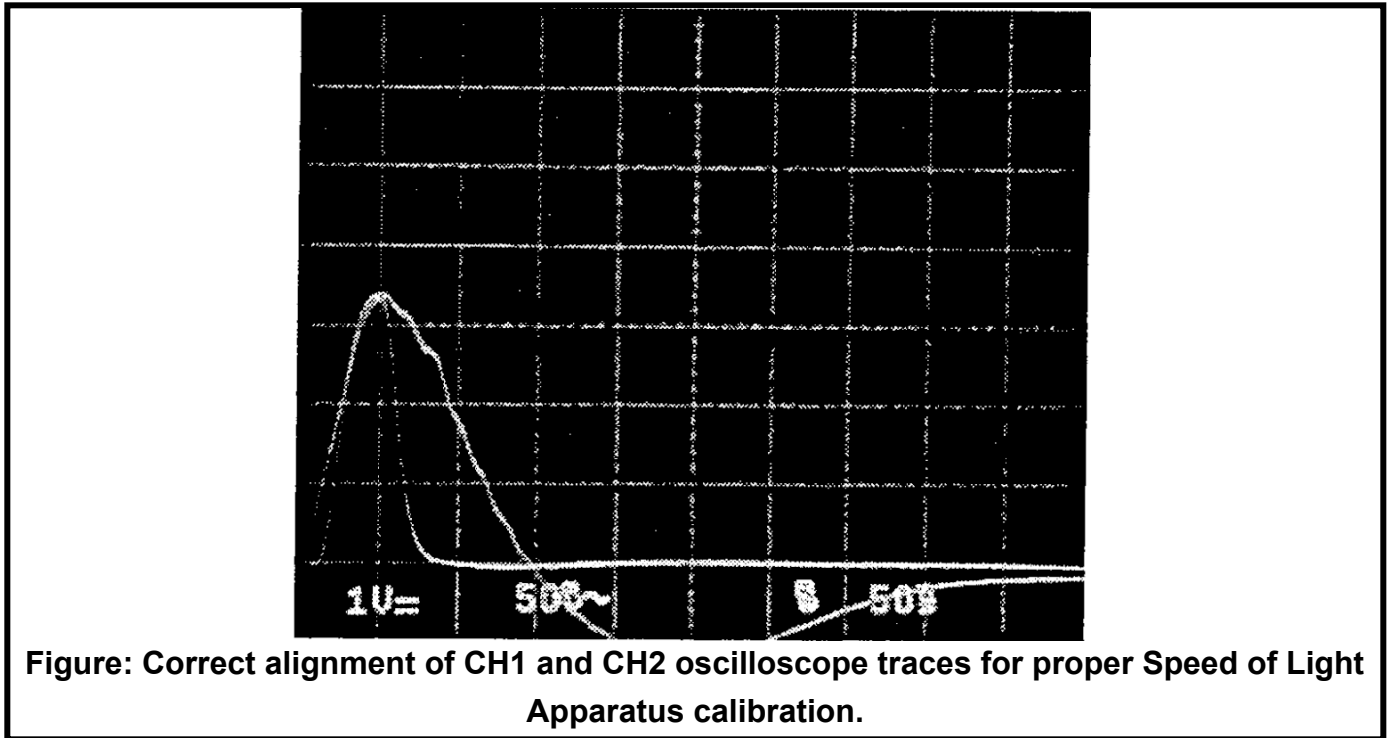
It is important to calibrate this apparatus to ensure accurate results for the remainder of the activity. The calibration will be done with 15 cm of optical fiber installed to simulate a distance of zero. Admittedly, 15 cm is not zero range, but the expected time delay in 15 cm of fiber is less than one nanosecond, which is not enough to affect the accuracy of this apparatus and test equipment. If at any time your results differ from those we describe, you should go back and double-check your measurements.

1. With the Speed of Light Apparatus and the oscilloscope on, a pulse should now be observed as CH1 (INPUT A) on the oscilloscope screen. It should be approximately 3.5 volts in amplitude and 35 nanoseconds in pulse width (full-width at half-maximum, FWHM). See below figure for reference. This is the calibration pulse which will serve as a reference pulse for subsequent measurements.



2. Switch CH2 (INPUT B) input coupling back from GND to AC.
3. A second pulse from 1 to 1.5 volts in amplitude and 75 nanoseconds wide (FWHM) should also now be visible. This is the pulse received through the 15-cm fiber optic cable.
4. Using the vertical positioning knobs, align the bases of CH1 and CH2 traces with the second division line above the bottom of the oscilloscope screen (as seen on above figure).
5. Using the horizontal positioning knob, align CH1 with the second division line from the left of the oscilloscope screen (as seen on above figure).

6. Rotate the **Calibration Delay** knob on the Speed of Light Apparatus until the peak of the received pulse coincides with the peak of the reference pulse as shown in below figure.



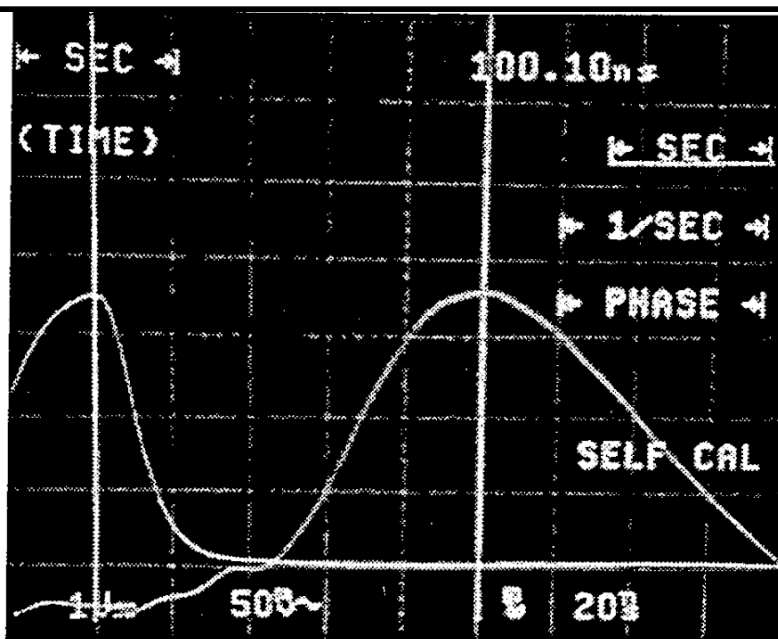
7. If necessary, turn on X5 MAG mode of the oscilloscope horizontal mode to enable best measurement of the signal delay time.
8. Fine-tune the Calibration Delay Adjustment knob on the apparatus to achieve best overlap/coincidence of the reference and received pulses.
9. With the calibration done, carefully loosen the fiber cinch nuts on LED **D3** and Detector **D8** and remove the 15 cm length of fiber.

## Measurement

Finally, we are going to make our speed-of-light measurement! Using the oscilloscope, we will measure the time required for the red LED light pulses to travel through 20 meters of plastic fiber.

1. Insert one end of the 20-meter plastic fiber gently but firmly, into LED **D3** until the fiber is seated. Lightly finger-tighten the fiber optic cinch nut on the LED.
2. Insert the other end of the fiber into Detector **D8** and lightly tighten the fiber optic cinch nut.

- Observe the display on the oscilloscope screen. The received pulse should now have moved to the right of the reference pulse, with a reduced amplitude of approximately 50%. See below figure as an example.



**Figure: Oscilloscope display showing approximate positions of the reference pulse and the received pulse delayed through 20 meters of fiber.**

- Very carefully measure the time difference between the reference pulse and the delay pulse, in nanoseconds. Make the measurement from peak of reference pulse to the peak of the delay pulse. As you are making the time-difference measurement, also estimate the uncertainty of your measurement (call me to ask, if you are not sure how to do that).

Your time measurement should be somewhere around 100 nanoseconds. If your measurement is very far from 100 nanoseconds, call me to help you double-check your measurement.

## Speed of Light Calculation

We now have just about all the information we need to calculate the speed of light. Because the light in our demonstration is traveling through a plastic fiber, which is not a vacuum, the speed of light measured in this lab will be less than the speed of light in vacuum,  $c = 3.00 \times 10^8 \text{ m/s}$ . But we can use the known value of vacuum speed of light and the measured speed of light in the transparent medium to calculate the index of refraction of the optical fiber we used.

- Calculate the speed of light in the optical fiber using the distance 20 meters and the delay time you measured above. Write down your result with the estimated uncertainties propagated (call me if you are not sure how to propagate uncertainties).
- Using the definition of index of refraction ( $n = c/v$ ), calculate the index of refraction of the plastic fiber. Write down your result with the estimated uncertainties propagated.

# Conclusion

Please make sure to write up the following on separate pieces of paper and turn them in:

1. A summary of procedure (***no more than 1 page in length***) for measurement of time delay and calculation of speed of light
2. Measurement of time delay ***with uncertainty estimates***
3. Calculation of speed of light in plastic fiber and index of refraction of fiber ***with propagated uncertainty from time delay measurement***
4. One-paragraph conclusion/summary for the lab



# Lab: Photoelectric Effect

This lab is adapted from UC Berkeley Physics 7C lab, "The Photoelectric Effect."

**Note:** For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on **separate pieces of paper to turn in**. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.

## Important Background Information About Photons and the Photoelectric Effect

Light is a wave. We showed this in two previous labs when we saw light interfering and diffracting, which can only be explained through a wave theory. We also can show it mathematically, using Maxwell's equations for the E- and B-fields in vacuum. In this lab, we will do an experiment demonstrating the **photoelectric effect**, in which light behaves like a particle. This surprising effect was first observed in 19th century, and it is Einstein's work of explaining this phenomenon with a hypothesis that light comes in discrete packets (particles) called **photons** that he was rewarded his Nobel prize in 1921.

## Photons

A particle of light is called a **photon**. Each photon in a beam of monochromatic light has an energy **E** and a momentum **p** that are given by

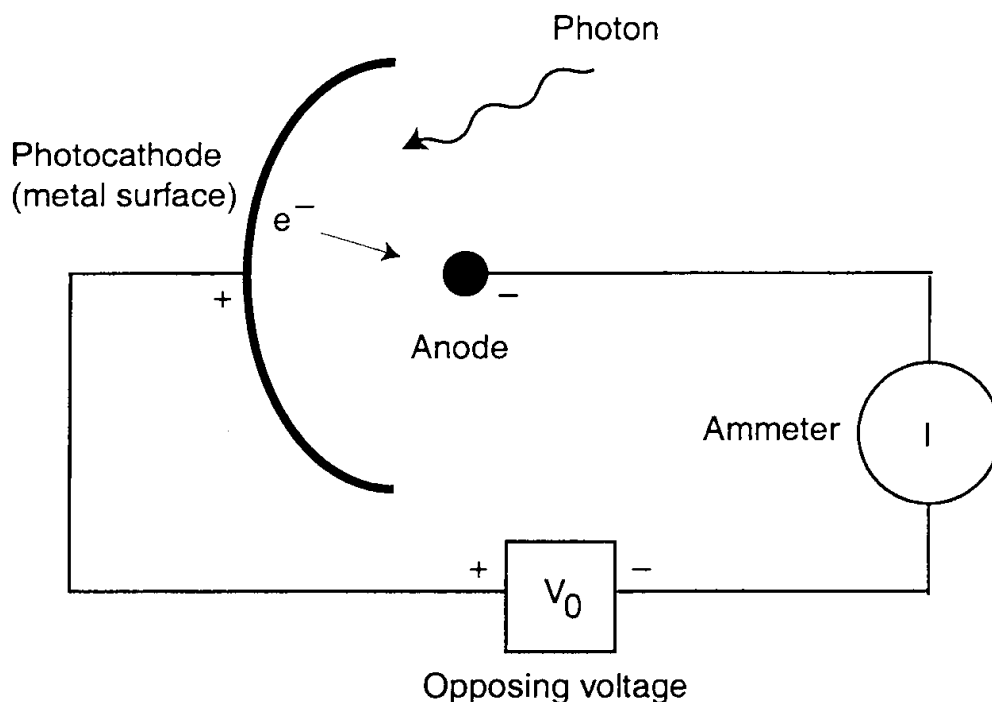
$$\begin{aligned} E &= hf \\ p &= \frac{E}{c} = \frac{hf}{c} \end{aligned} \quad \text{Eq. 1}$$

where **c** is the speed of light in a vacuum, **f** is the frequency of the light beam, and **h = 6.63 x 10<sup>-34</sup> J s** is known as **Planck's constant** (more conveniently expressed in electron-volt units: **h = 4.14 x 10<sup>-15</sup> eV s**).

When light is absorbed by a material, it is absorbed photon by photon. Thus the material absorbs energy from the light in integer packets of **E = hf**, and momentum in packets of **p = hf/c**. The light cannot give up its energy and momentum in any other amounts. This **quantization** of energy and momentum is completely different from the classical wave model of light, in which a material can absorb any amount of an incident light wave's energy and momentum, depending on the index of refraction, angle of incidence, and other wave parameters.

# The Photoelectric Effect

The experiment proving Einstein's photon hypothesis was first done in 1914 by R. A. Millikan, which is what we are reproducing in our lab. In this experiment, a beam of light shines on a metal surface (we call it **photocathode**, for historical reasons), and some of the energy of the incident light is absorbed by electrons in the metal. The electrons use this energy they absorb to try to escape the potential that binds them to the metal. One can measure the number of electrons liberated by the light by collecting them on a piece of metal (that we call **anode**, also for historical reasons), and measuring the resulting current, called **photocurrent**, that passes through the anode. To collect all of the electrons that are liberated, one would give the anode a positive potential so that the electrons would be attracted there (you might remember from chemistry that, given a battery, "cathode" is the negatively charged end and "anode" is the positively charged end, which is the historical reason for the names we give above). Figure 1 below illustrates this arrangement schematically.



**Figure 1:** The setup of the photoelectric effect experiment. When an electron escapes from the photocathode with too little kinetic energy to overcome the opposing voltage  $V_0$ , it can't reach the anode and doesn't contribute to the measured photocurrent.

For this experiment, however, we are not interested in the total number of electrons liberated. Instead, we are interested in what their kinetic energy (**KE**) is when they escape. By energy conservation, **KE** is equal to the energy the electrons absorb from the light, minus the energy it takes to liberate them (their binding energy). We can measure this by varying the potential applied to the anode. When the anode potential is positive the electrons are attracted there, and so we see a large photocurrent (up to the maximum of number of electrons liberated from photocathode). If we drop the anode voltage to zero, the electrons are neither attracted nor repelled, but those that happen to leave the metal with a velocity toward the anode still get there. If we now apply a small *negative* voltage to



the anode, the electrons are repelled, but some heading directly towards the anode may *still* reach it if they have sufficient kinetic energy. You can think of this like a ball rolling up a hill. They'll make it up the hill, if their kinetic energy at the bottom is greater than or equal to their potential energy at the top. By gradually lowering the potential of the anode (applying an increasingly negative potential), we can find the **stopping potential**  $V_{\text{stop}}$  at which no electrons reach the anode and the photocurrent drops to zero. The electrons' kinetic energy **KE** is therefore equal to their potential energy at a potential  $V_{\text{stop}}$ .

In fact, the electrons do not all leave the metal with the same kinetic energy, for they can be bound to the metal with different binding energies. Those that are tightly bound (with a large binding energy) will escape with little kinetic energy; those that are least tightly bound (with the smallest binding energy) escape with the most kinetic energy, which we will call  $\text{KE}_{\text{max}}$ . Thus as we raise the anode potential to nearly  $V_{\text{stop}}$ , only electrons with  $\text{KE}_{\text{max}}$  can still reach the anode. So it is really  $\text{KE}_{\text{max}}$  that is equal to the potential energy of an electron at a potential  $V_{\text{stop}}$ :

$$eV_{\text{stop}} = \text{KE}_{\text{max}}. \quad \text{Eq. 2}$$

Remember that the potential energy of an electron at a voltage  $V$  is  $eV$ . By measuring  $V_{\text{stop}}$ , we can determine  $\text{KE}_{\text{max}}$ .

The photoelectric experiment examines how  $\text{KE}_{\text{max}}$  varies with the intensity and frequency of the incident light. This allows us to explore two possibilities for the interaction of light with matter: does light interact with matter (1) as a wave, or (2) as a particle?

If light interacts with matter as a wave, it should give up its energy smoothly and continuously. Electrons absorbing high-intensity light would get a lot of energy very quickly and rapidly break free of their potential. Electrons that absorb low-intensity light, on the other hand, would gradually gain energy and break free of the metal only after some time. Therefore there might be a time lag between when the light beam is turned on and when the photocurrent starts to flow, depending on the light intensity (the theoretical estimate for this time lag varies, depending on what you consider to be effective area of the electron and rate at which the electron gathers energy from light). The light's frequency should be irrelevant. The dependence of  $\text{KE}_{\text{max}}$  on light's intensity or frequency would be harder to determine theoretically, but we might expect it should depend more on intensity, since intensity of light is related to the amplitude of electric field, which applies force on the electrons (frequency merely tells you how often the direction of this force switches back and forth).

However, what was observed in photoelectric effect since early 20th century was completely different from this expectation based on interaction of light with matter as a wave. First of all, there is no time lag between when the beam was turned on and when the photocurrent began that depended on incident intensity of light. Second, while the *amount* of photocurrent depends on the light's intensity,  $\text{KE}_{\text{max}}$  does not. Instead,  $\text{KE}_{\text{max}}$  depends on *frequency*: the higher the frequency of the light, the higher  $\text{KE}_{\text{max}}$  of the liberated electrons. And below a certain frequency, called the **cutoff frequency**  $f_0$ , the photocurrent drops to zero for all values of the (positive) anode voltage, no matter how high

the incident intensity. This phenomenon of frequency-dependent  $\mathbf{KE_{max}}$  and no time-lag was named the **photoelectric effect**. The wave model of light's interaction with matter can't explain it.

However, the photon model of light *can* explain the photoelectric effect. If light interacts with matter as photons—that is, as particles rather than as a wave—then an electron would absorb the first photon that struck it, just as the beam was turned on. Upon absorbing a single photon, the electron gains  $\mathbf{E=hf}$  worth of additional energy (no more, no less). If this is greater than the potential energy needs to break free of the metal, the electron escapes with a kinetic energy given by

$$\mathbf{KE \leq KE_{max} = hf - \phi} \quad \mathbf{Eq. 3}$$

where  $\phi$ , called the work function, is the potential energy that binds the *least* tightly bound electrons. Notice that this model (considering interaction of light with electron as collision between two particles) shows that  $\mathbf{KE_{max}}$  depends on the frequency of the light  $\mathbf{f}$  but not on the light's intensity. In addition, a cutoff frequency  $\mathbf{f_0}$  is predicted,

$$\mathbf{f_0 = \frac{\phi}{h}} \quad \mathbf{Eq. 4}$$

where even the least tightly bound electrons don't receive enough energy from a single photon to break free. And since electrons bound in the metal are in constant interaction with surrounding particles (other electrons and atomic nuclei), it would not retain this additional energy for long enough for a second photon to strike while it still has the additional energy from the first photon. So no electrons can leave the photocathode in this case and the photocurrent is zero.

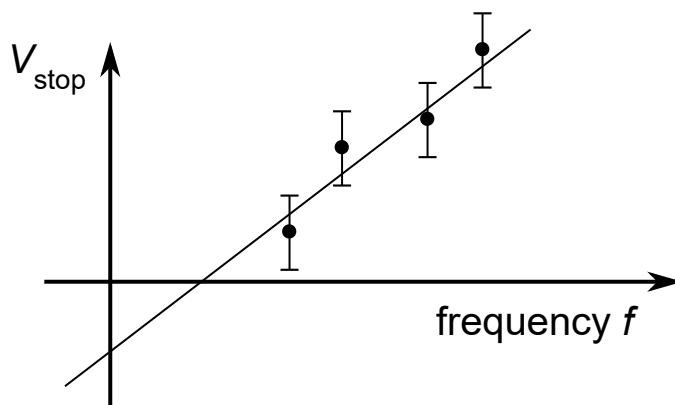
The photoelectric effect demonstrates that light behaves like a particle. But our diffraction and interference experiments demonstrate that it also behaves like a wave. Even though our wave theory is being thrown into some doubt at the moment, those diffraction and interference *experiments* were not wrong. The idea that light is somehow both a particle and a wave is called the **wave-particle duality of light**. It is not an intuitive idea, since in classical mechanics everything gets placed into an "either-or" category of particles (electrons, protons, neutrons, baseballs, etc.) or waves (light, sound, vibrations, etc.), not into both. But this is certainly a ubiquitous quantum phenomenon, and it is something you should start developing your intuition around. If it helps, you can take the following statement as being true (as far as we know so far): **"Light travels as a wave but interacts with matter as a particle."**

# How This Experiment Works

We're going to demonstrate the photoelectric effect for ourselves, proving that light interacts with matter as a particle. We'll also measure Planck's constant  $h$ , a fundamental quantity in quantum mechanics, and the work function  $\phi$  of the metal we are using for our photocathode. We do this by first realizing that if we combine Equations 2 and 3 we obtain

$$KE_{\max} = eV_{\text{stop}} = hf - \phi. \quad \text{Eq. 5}$$

This is a linear function that relates  $V_{\text{stop}}$  and  $f$ . By measuring the stopping potential,  $V_{\text{stop}}$ , for various values of the frequency of incident light  $f$ , we can fit a line to the points and calculate  $h$  and  $\phi$  (see Figure 2 below).



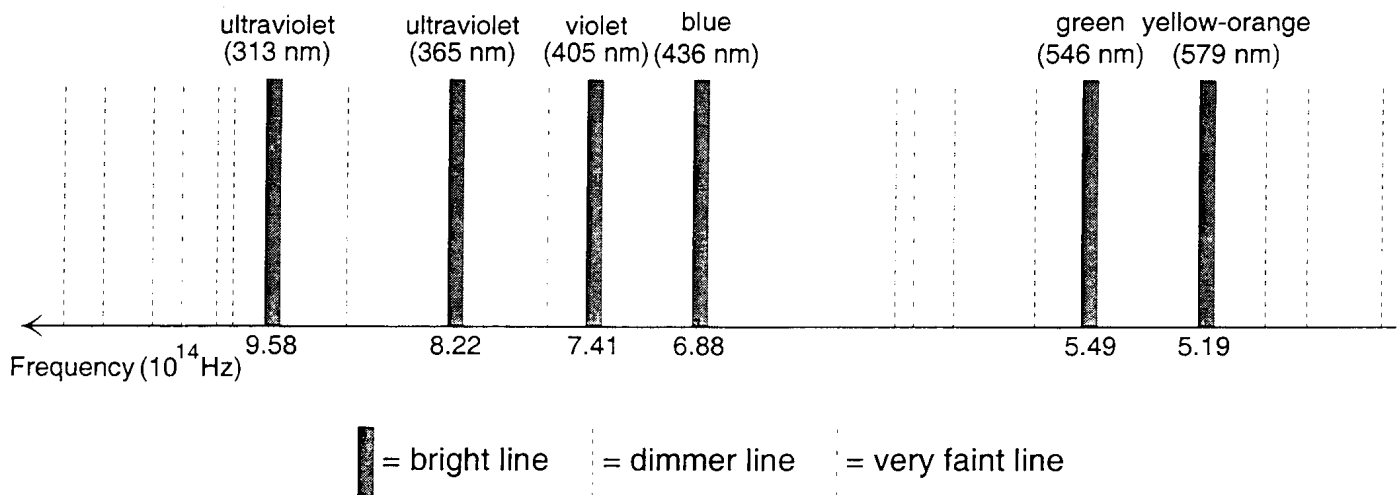
**Figure 2:** A plot of data points typical in the photoelectric effect experiment, and the line fit to them. The slope and  $y$ -intercept of this line can be used to determine  $h$  and  $\phi$ .

To do this we need a light source with different frequencies, a photocathode on which to shine the light (and an anode positioned in a way that photoelectrons can reach), a voltage source to supply the voltage  $V_{\text{stop}}$ , and a way to measure the photocurrent from the photocathode.

## The Mercury Lamp Light Source

We obtain our different frequencies of light from a mercury lamp and interference filters. The light from the mercury lamp is a combination of many different frequencies, and it appears in blue-to-white color to our eyes. You can, however, see that this light is made up of distinct frequencies (spectral lines) by looking at the mercury lamp through a diffraction grating. When you look at the mercury lamp through a diffraction grating, the grating disperses the different frequencies of light in different directions, just as you saw in [Lab 3: Diffraction and Interference](https://peralta.instructure.com/courses/51133/pages/lab-3-diffraction-and-interference) (<https://peralta.instructure.com/courses/51133/pages/lab-3-diffraction-and-interference>) with green and red lasers. For this lab, we are going to use interference filters (specially made optical filters which causes destructive interference for most frequencies of light except for a small range they are designed to select) to pick out a particular spectral line for each measurement. The figure below lists the colors you should see in the spectrum (if you look at it through a diffraction grating), and some of which our interference filters are designed to select.

### Mercury Lamp Spectral lines



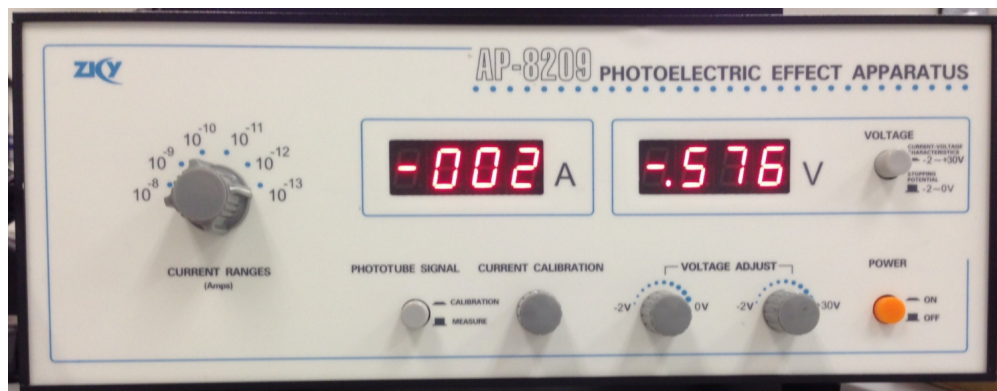
**Figure 3:** Strong spectral lines in the mercury spectrum. Frequencies and wavelengths of the bright lines are given in the figure.

## The Phototube

Our photocathode and anode are inside a commercially-purchased glass vacuum tube, called a phototube. One phototube box will be open for you to be able to see what is inside the black box in your lab setup (schematically, it is what is shown in Figure 1).

## The Control Box

The current we are trying to measure is tiny, in the range below 10 nanoamps (1 nanoamp is  $10^{-9}$  amp), so we need an extremely sensitive ammeter. This extremely sensitive ammeter has been integrated into the control box, shown in Figure 4 below. This control box also integrates the voltage you will be applying to the anode. Please take a look at the demonstration setup with the open phototube blackbox to verify the electrical connections.



**Figure 4:** Front panel of the Photoelectric Effect Apparatus control box.

# Procedure

Start by verifying the connection between different parts of the apparatus. The power supply powers both the mercury lamp and the control box, and the control box provides voltage to the phototube.

Turn on the mercury lamp and look at the lamp light through the provided high-quality diffraction grating to verify that the mercury spectrum looks as described in Figure 3. You may need to remove the phototube black box from the mount in order to get a good look at the mercury lamp (please be careful with the black box; its contents are fragile). Leave the mercury lamp for the duration of the lab, as it takes some time to warm up fully. *[Note: You may ignore the warnings about covering the mercury lamp; I am not sure why that warning is there, as the mercury lamp light is not harmful to you or the phototube, except possibly by causing an excess current which could conceivably burn out the ammeter, if it does not have built-in protection circuits. I will remove this note the first time an ammeter burns out from excess current.]*

**Q1:** Sketch the spectrum of mercury lamp as seen through the high-quality diffraction grating, clearly labeling the red and the blue side of the spectrum. Make a note of relative intensities of different lines seen.

Prepare for the current measurement by calibrating the zero of current, following these steps:

1. Set CURRENT RANGES to  $10^{-13}$  (most sensitive scale).
2. Disconnect 'A', 'K', and 'down arrow' (GROUND) cables from the back panel of the control box (this ensures that actual current flowing is zero).
3. Press the PHOTOTUBE SIGNAL button in to CALIBRATION.
4. Adjust the CURRENT CALIBRATION knob until the current is zero. This calibrates the zero point.
5. When the calibration is done, press the PHOTOTUBE SIGNAL button to MEASURE and reconnect 'A', 'K', and 'down arrow' (GROUND) cables that you disconnected in Step 2.

Place an interference filter (pick a wavelength—if you aren't sure, try 405 nm) for the mercury lamp. Place the phototube black box as close to the mercury lamp as you can (for maximum intensity), and vary the applied voltage on the control box, watching for the change in the photocurrent.

**Q2:** Describe how the photocurrent changes as a function of applied voltage. Based on your understanding of the setup, does the change in photocurrent make sense? Note anything unusual you see and explain what you can. Call me if there are any questions.

**Q3:** Using the controls on the control box, determine  $V_{\text{stop}}$  for this wavelength of light. In your judgment, approximately how accurately can you determine  $V_{\text{stop}}$ ? Give your estimate in the form of  $\pm \Delta V$  (where  $\Delta$  is your uncertainty in the unit of volts), and explain how you estimated  $\Delta$ .

Switch out the filter for a different wavelength and measure  $V_{\text{stop}}$  for each of the wavelength listed in Figure 3, verifying the error estimate with each measurement. For each measurement, record the frequency also, for the analysis questions.

# Analysis

**Q4:** Plot your values of  $V_{\text{stop}}$  vs. frequency as in Figure 2, including the error bars you decide on in Q3 above. Draw your best-fit line through your points and determine its slope,  $y$ -intercept, and  $x$ -intercept (you may use a calculator's linear regression function to determine these best-fit line parameters).

**Q5:** Using the values from your best-fit line, what is your measurement of  $h$ ? How close is this to the given value? What is your measurement of  $\phi$ ? Of the cutoff frequency  $f_0$ ? Give an estimate of uncertainty for each value (your best-fit function might yield an uncertainty on the parameters; if not, you can manually estimate uncertainty based on error bars—call me if you aren't sure how to do that).

**Q6:** Make a note on the uncertainty and error (note, these are two different things) you have calculated in Q5 for Planck's constant  $h$ . What are possible sources of error? How does each source of error affect the measured value of  $h$ ? You might want to look at the photocathode setup (using the demonstration setup with exposed photocathode) to help you think through possible sources of error.

**Q7:** A table of work function by elements is listed below. Based on the work function  $\phi$  you measured, what are possible candidates for your photocathode (assuming that your photocathode is an elemental metal)? Time-permitting, do a brief search on the candidate materials to see if the photocathode you see (use the demonstration setup if needed) could be the candidate material.

# ELECTRONIC WORK FUNCTIONS OF THE ELEMENTS

compiled by Herbert B. Michaelson (From CRC)

Abbreviations apply to experimental method: T, thermionic; P, photoelectric;  
CPD, contact potential difference; F, field emission.

| Element | Experimental value, $\phi$ (ev)  | Experimental method        | Element    | Experimental value, $\phi$ (ev)   | Experimental method                  | Element | Experimental value, $\phi$ (ev)  | Experimental method                      |
|---------|--|----------------------------|------------|---|--------------------------------------|---------|--|--|
| Ag      | 4.26<br>4.64 (100)<br>4.52 (110)<br>4.74 (111)   | P<br>P<br>P<br>P           | Hf         | 3.9   | P                                    | Re      | 4.96<br>5.75(1011)   | T<br>F                                   |
|         |  |                            | Hg         | 4.49  | P                                    | Rh      | 4.98   | P  |
|         |  |                            | In         | 4.12  | P                                    | Ru      | 4.71   | P  |
| Al      | 4.28<br>4.41 (100)<br>4.06 (110)<br>4.24 (111)   | P<br>P<br>P<br>P           | Ir         | 5.27<br>5.42 (110)<br>5.76 (111)<br>5.67 (100)<br>5.00 (210)  | T<br>F<br>F<br>F<br>F                | Sb      | 4.55 (amorph.)<br>4.7 (100)  | -<br>-                                   |
| As      | 3.75   | P                          | K          | 2.30  | P                                    | Sc      | 3.5  | P  |
| Au      | 5.1<br>5.47 (100)<br>5.37 (110)<br>5.31 (111)  | P<br>P<br><br>             | La         | 3.5   | P                                    | Se      | 5.9  | P  |
|         |  |                            | Li         | 2.9   | F                                    | Si      | 4.85n<br>4.91p (100)<br>4.60p (111)  | CPD<br>CPD<br>P                          |
| B       | 4.45   | T                          | Lu         | 3.3   | CPD                                  | Sm      | 2.7  | P  |
| Ba      | 2.7  | T                          | Mg         | 3.66  | P                                    | Sn      | 4.42   | CPD                                      |
| Be      | 4.98   | P                          | Mn         | 4.1   | P                                    | Sr      | 2.59   | T  |
| Bi      | 4.22   | P                          | Mo         | 4.6<br>4.53 (100)<br>4.95 (110)<br>4.55 (111)<br>4.36 (112)<br>4.50 (114)                             | P<br>P<br>P<br>P<br>P<br>P           | Ta      | 4.25<br>4.15 (100)<br>4.80 (110)<br>4.00 (111)                             | T<br>T<br>T<br>T                         |
| C       | 5.0  | CPD                        |            |   |                                      | Tb      | 3.0  | P  |
| Ca      | 2.87   | P                          |            |   |                                      | Te      | 4.95   | P  |
| Cd      | 4.22   | CPD                        | 4.55 (332) | P   |                                      | Th      | 3.4  | T  |
| Ce      | 2.9  | P                          | Na         | 2.75  | P                                    | Ti      | 4.33   | P  |
| Co      | 5.0  | P                          | Nb         | 4.3<br>4.02 (001)<br>4.87 (110)<br>4.36 (111)<br>4.63 (112)<br>4.29 (113)<br>3.95 (116)<br>4.18 (310) | P<br>T<br>T<br>T<br>T<br>T<br>T<br>T | Tl      | 3.84   | CPD                                      |
| Cr      | 4.5  | P                          |            |   |                                      | U       | 3.63<br>3.73 (100)<br>3.90 (110)<br>3.67 (113)                             | P & CPD<br>P & CPD<br>P & CPD<br>P & CPD |
| Cs      | 2.14   | P                          |            |   |                                      | V       | 4.3  | P  |
| Cu      | 4.65<br>4.59 (100)<br>4.48 (110)<br>4.94 (111)<br>4.53 (112)                               | P<br>P<br>P<br>P<br>P      | Nd         | 3.2   | P                                    | W       | 4.55<br>4.63 (100)<br>5.25 (110)<br>4.47 (111)<br>4.18 (113)<br>4.30 (116) | CPD<br>F<br>F<br>F<br>CPD<br>T           |
| Eu      | 2.5  | P                          | Ni         | 5.15<br>5.22 (100)<br>5.04 (110)<br>5.35 (111)  | P<br>P<br>P<br>P                     | Y       | 3.1  | P  |
| Fe      | 4.5<br>4.67 (100)<br>4.81 $\alpha$ (111)<br>4.70 $\alpha$<br>4.62 $\beta$<br>4.68 $\gamma$ | P<br>P<br>P<br>P<br>P<br>P | Os         | 4.83  | T                                    | Zn      | 4.33<br>4.9 (001)  | P<br>CPD                                 |
|         |  |                            | Pb         | 4.25  | P                                    | Zr      | 4.05   | P  |
| Ga      | 4.2  | CPD                        | Pd         | 5.12<br>5.6 (111)   | P<br>P                               |         |  |  |
| Ge      | 5.0<br>4.80  | CPD<br>P                   | Pt         | 5.65<br>5.7 (111)   | P<br>P                               |         |  |  |
| Gd      | 3.1  | P                          | Rb         | 2.16  | P                                    |         |  |  |





# Lab: Quantum Mechanics Simulations


**Note:** For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on **separate pieces of paper to turn in**. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.

## Introduction

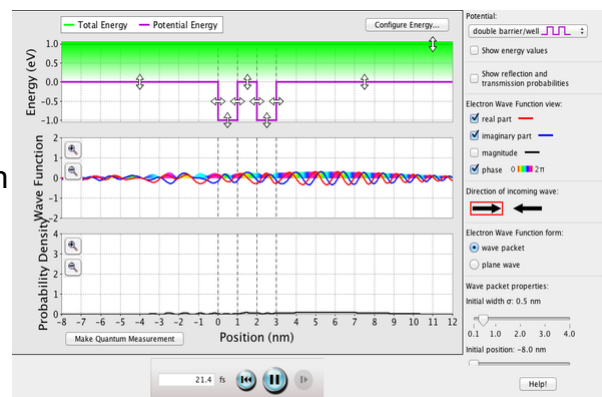
By now, we have been discussing experimental results that led to the development of quantum mechanics, some of the key assumptions (initially *ad hoc*, later more formalized) that form the starting points for quantum mechanics. There are two main difficulties in correctly understanding quantum mechanics beyond simply memorizing some formulas, like the energy levels in the Bohr model: (1) No one comes in with any level of intuitive understanding for quantum mechanics. Your intuition is based on situations you have encountered and have some experience with. Because the quantum mechanical effects most clearly become manifest in microscopic (or even subatomic) situations, your everyday experience did not help you develop any intuition for quantum mechanics. (2) The mathematics of quantum mechanics is challenging, involving fair amount of calculus, and at times, linear algebra (the kind with complex linear vector space).

Our goal in this lab is to help you develop some intuition for the *wave mechanics* formulation of quantum mechanics, exploring the consequences of the de Broglie hypothesis, that everything that has a momentum has a wavelength, and that they are related by the expression,  $p = h/\lambda$ . We will use numerical computer simulations to help bypass the issue (2) for now. (I believe it's the job of your upper-division quantum-mechanics class to teach you how to actually handle the math involved.)

## Part A: Wave Packet

You will use "[Quantum Tunneling](https://phet.colorado.edu/en/simulation/legacy/quantum-tunneling)" PhET simulation  (<https://phet.colorado.edu/en/simulation/legacy/quantum-tunneling>) to explore properties of matter wave, how it interacts with external potential, and how it responds when you make a measurement of particle position.

First, download the Java simulation and run it on your computer. Explore the simulation on your own for a while, until you feel comfortable with its features. Ask me any questions about different aspects of the simulation that you don't feel that you understand. When you



feel comfortable with the simulation (perhaps after 5 to 10 minutes), proceed with the instructions and questions below.

First, set up the simulation for the initial condition as following:

- Choose Potential to be constant at 0 eV.
- Choose Electron Wave Function view that you are comfortable with. If you have no preference, I recommend selecting “magnitude” and “phase” (but not “real part” or “imaginary part”).
- After pausing the simulation, select the initial width and position of the wave. Select initial width of 1.0 nm, and choose the initial position as far to the left as possible that allows you to see the whole wave packet.

You are going to measure the uncertainty in the momentum of the wave packet, and this is how you will do it:

1. Note the initial position of the wave packet, and let the simulation run until the wave packet has almost reached the right edge of the screen. Pause the simulation.
2. Measure the distance that the peak of wave packet has traveled. Use the time measurement of the simulation to estimate the average speed and average momentum of the wave packet.
3. You should have seen the wave packet spreading noticeably. Use the spreading of the wave packet to estimate the uncertainty in momentum. That is, calculate a *range* of momenta contained within the initial wave packet.

Look up any physical constants needed. Please answer the questions below and follow further instructions.

**Q1:** What are the uncertainty in position ( $\Delta x = \sigma$ ) and momentum of the wave packet at the initial condition? Is this consistent with Heisenberg uncertainty principle? Explain your answer.

**Q2:** Make a measurement of the uncertainty in position when you paused the simulation (you will need to measure new width of the wave packet). Is this new value consistent with Heisenberg uncertainty principle? Explain your answer.

**Q3:** You can prepare a particle (i.e. wave packet) initial state in such a way that the initial uncertainty in position is small. However, the principles of wave mechanics says that, for a free particle, the uncertainty in position *does not remain* small. Explain conceptually why this is the case.

You can set up the simulation with a potential to reflect an incoming wave. Set up the simulation (after pausing it) as following for the next question:

- Choose Potential to be “step.”
- Set the energy values so that the energy before the step is -1.0 eV; the energy after the step is 0.5 eV, and the energy of the incoming particle is -0.5 eV.
- Set the initial width of the particle to be 1.0 nm, with the initial position as far to the left of the screen as possible.

Run the simulation, you will see the wave packet bounce off from the step. Repeat the simulation as many times as necessary until you feel you have an understanding of what happened.

**Q4:** Does what you see agree with your intuition (which, mind you, is built up using Newtonian mechanics)? Explain how what you saw makes sense.

Lower the potential after the step to *below* -0.5 eV. Try running the simulation again and see what happens. (The sudden change in potential causes a portion of the wave to reflect off; it doesn't really correspond to behavior of macroscopic objects, such as an airplane approaching and going over a cliff, but we can find classical waves, such as electromagnetic waves, that behave the same way.)

Now, set up the simulation with a barrier/well potential. We will observe a phenomenon called "quantum tunneling." Set up the simulation as below:

- Choose Potential to be "barrier/well".
- Set the energy values so that the energy before the barrier is -1.0 eV; the energy *during* the barrier is 0.5 eV; and the energy *after* the barrier is back to -1.0 eV. Leave the width of the barrier at the default value, you will be changing that in answering questions below.
- Leave the particle state same as before.

Run the simulation, observing what happens with the wave packet. Repeat it as many times as necessary to understand what is going on.

**Q5:** Describe what happened with the wave packet. Does it agree with your intuition? Does it agree with your previous result?

During the interaction, you should have seen a significant portion of the particle wave packet enter into the barrier. So what you will do now is decrease the width of the barrier, so that the portion of the particle wave packet that enters into the barrier extends to the *other side* of the barrier. Run the simulation, observing what happens with the wave packet.

**Q6:** Describe what happens now. Explain what you would have expected to happen in classical mechanics (and why). Explain how what you see with *wave mechanics* is different (and why).

**Q7:** Find ("experimentally") the width of the barrier that leads to 10% tunneling probability through the barrier. Give the width of the barrier in units of nanometers.

Reset the potential back to "constant," and increase the particle energy back up to 0.5 eV. We are going to look at the effect of "quantum measurement."

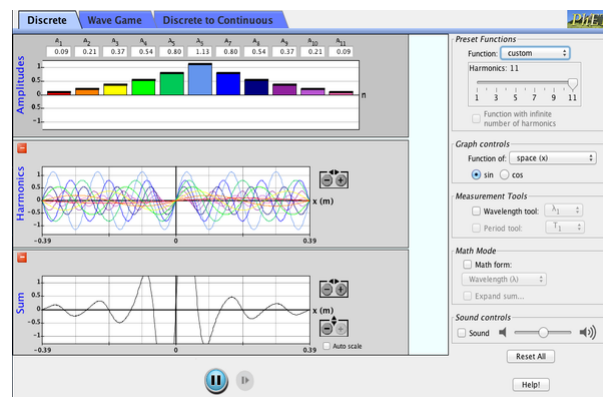
**Q8:** Pause the simulation, and click on "Make Quantum Measurement" (it makes measurement of particle position). Observe what happens to the wave function and the probability density. Does what you see make sense, based on what you know of electron (particle represented by the wave function), and what measurement does? Explain.

**Q9:** Unpause the simulation and observe what happens. Explain why what you observe happens.

One of the classical notion that we have to give up in quantum mechanics is the idea that you can make a measurement of a particle's property (such as position and momentum) without affecting the state of the particle itself. In classical mechanics, it was an article of faith that you could make any disturbance due to a measurement as small as possible, so that the disturbance, in theory, was zero. In quantum mechanics, we admit to the fundamental limitation on how small we can make this disturbance due to a measurement, so the theoretical description of the particle state *includes* this fundamental uncertainty, and this leads to Heisenberg uncertainty principle. There are other formulations of quantum mechanics ("matrix mechanics") that start out with this uncertainty principle as the founding assumption.

## Part B: Fourier Analysis

The position-momentum uncertainty principle can be understood as a *natural* consequence of position-wavelength uncertainty that already existed in classical mechanics (dealing with water waves, EM waves, sound waves, etc.). That is, once you accepted that a particle has wave property (through de Broglie hypothesis), then you *should have expected* this relationship between position and momentum uncertainty. The mathematical description of this position-wavelength uncertainty comes from Fourier analysis, and we will use the ["Fourier" PhET simulation](https://phet.colorado.edu/en/simulation/legacy/fourier)



(<https://phet.colorado.edu/en/simulation/legacy/fourier>) to help us explore some of the consequences (without actually having to do the integrals and other math involved in actual Fourier analysis).

Download the Java simulation and run it on your computer. Explore the simulation on your own, until you feel comfortable with its features. Make sure to toggle on "Math form" in the "Math Mode". This will display the mathematical expressions so that you can get into more detail, if you wish.

Play with different preset functions, and observe how different waveforms, such as square waves and triangular waves are formed by combining sinusoidal waves of different harmonics. When you are ready, proceed to the question below.

**Q10:** Choose "wave packet" as Preset Function. This will set up the harmonics so that you see a wave packet on the bottom screen (you may need to zoom out in the y axis). Toggle between "sin" and "cos" to see either odd or even wave packet. Describe how the wave packet is formed by superposition of all the harmonics selected. In particular, explain what is happening at the location where wave packet is at maximum amplitude (easier to visualize with cosines), and what is happening at the location where the wave packet sum is zero.

**Q11:** Under "Graph controls," choose the wave to be a function of space *and* time. Observe what happens. Explain how it makes sense. (You can pause the simulation to stop the flow of time in the simulation, if you wish).

The wave packet you see above does not faithfully represent a single particle, mainly because the wave packet is periodic (you saw that in Q11). To obtain something that would represent a single particle, we have to go to the continuous limit. Switch to the “Discrete to Continuous” tab.

Play with the simulation in this tab until you feel comfortable with different features here. Proceed when ready.

Set up the simulation for the initial condition by clicking “Reset All.” Answer the questions below. Click “Reset All” between questions unless directed otherwise.

**Q12:** Describe what you see when you change the “Wave packet center.” The value of  $k_0$  there represents the mean wavenumber (related to mean momentum by  $p = \hbar k$ , in quantum mechanics). Observe how Amplitudes plot and Sum plot changes as you vary the wave packet center. Do the changes make sense?

**Q13:** Try changing the Wave packet width. The two sliders correspond to wave packet width in two different “spaces.” The first slider affects the wave packet width in the “ $k$  space” (also known as “momentum space” in quantum mechanics). The second slider affects the wave packet width in the “ $x$  space,” or “position space”. Describe what you see as you make the wave packet narrower either in  $k$  space or  $x$  space. Explain if the wave packet changes the way you expected. Also explain how wave packet width in  $k$  space is related to wave packet width in  $x$  space.

What you observed in Q13 is the fundamental reason behind the Heisenberg uncertainty principle (once you accept de Broglie hypothesis): you cannot make the wave packet narrower in position space without also making it wider in the momentum space. This connection is so intimate, that it works backward as well: there are formulations of quantum mechanics (to be covered in upper-division courses) where you *start* from Heisenberg uncertainty principle (as your fundamental, axiomatic assumption), and *derive* de Broglie relationship, rather than assuming it to start (“matrix mechanics”).

So far, you have dealt with periodic wave packets. Now we will see what is required (mathematically) to represent a single, spatially localized particle. First, zoom out on Sum graph as far as possible. Reduce the “Spacing between Fourier components.” Do it step by step (there are total of 3 steps), observing what happens with each step.

**Q14:** “Continuous limit” refers to when the spacing between Fourier components is zero. That is, the wave packet contains *all* the frequencies, within the envelope described by the wave packet width in  $k$  space. Describe what is required to mathematically represent a single particle localized within some space width width of  $\sigma_x$ .

**Q15:** There exist particles known as “point particles” (one you know best is an electron). These are particles, as far as we know, do not have a finite size. As far as we can determine experimentally, these particles have no measurable size (i.e. no “radius of electron”), so we ought to be able to describe such point particle with a wave function of infinitesimally small width in position space. Describe some of the properties of such wave function. (For example, what frequency components does such a wave function contain?)

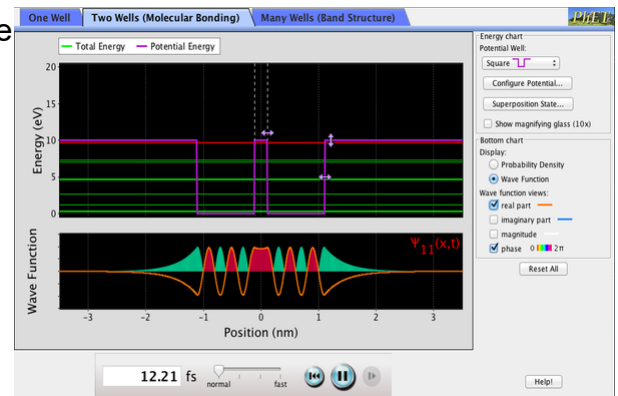
The “function” you described above is known as “Dirac delta function,” and you will learn more about it, if you take upper-division quantum mechanics.

## Part C: Bound States

If you finished all the questions above and have some time left, you can explore the ["Bound States"](https://phet.colorado.edu/en/simulation/legacy/bound-states)

[PhET simulation](https://phet.colorado.edu/en/simulation/legacy/bound-states)

(<https://phet.colorado.edu/en/simulation/legacy/bound-states>). This simulation simulates energy eigenstates in finite-well and Coulomb-type potentials. The double-well potential particularly is able to demonstrate the difference between states referred to as “stationary state” and “coherent superposition.”



If you have time to explore these, please call me and I will show you what settings to play with.

# Lab: Atomic Spectra

This lab is adapted from UC Berkeley Physics 7C lab, "Atomic Spectra."

**Note:** For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on **separate pieces of paper to turn in**. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.

## Important Background Information About Atomic Spectra

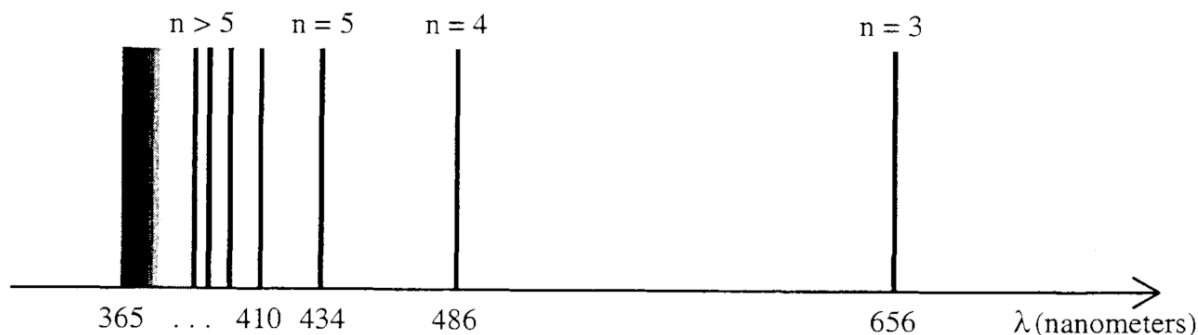
All normal matter is made of atoms. One of the best ways of learning about atoms is to study the light they emit when excited by an electrical current. The interesting thing about this light is that it only contains certain specific wavelengths. The set of these wavelengths is called the **spectrum** of the atom, and is unique for each type of atom.

Scientists have known experimentally the spectrum of hydrogen, the most abundant element, for a long time (see Figure 1). When hydrogen atoms are excited by electricity (or any other means) the light they emit contains wavelengths given by the **Balmer-Rydberg formula**

$$\frac{1}{\lambda} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right) \quad \text{Eq. 1}$$

$$R = 1.097 \times 10^7 \text{ m}^{-1}$$

where  $m$  and  $n$  are any two positive integers with  $m < n$ , and  $R$  is called **Rydberg's constant**. Notice that since  $n$  and  $m$  can be infinitely high, there is an infinite number of wavelengths in the hydrogen spectrum. However, the spectrum is still made up of distinct wavelengths and the wavelengths in-between are not included.



**Figure 1:** The spectrum of hydrogen, in the region  $360 \text{ nm} \leq \lambda \leq 700 \text{ nm}$  (visible light). The wavelengths in this region correspond to  $m = 2$  and  $n = 3, 4, 5$ , etc. Each vertical wavelength stripe is called a **line**. For high values of  $n$ , the lines are hard to distinguish.

One of the most important goals of early atomic theory was to create a model of hydrogen that could explain why its spectrum obeys the Balmer-Rydberg formula. The first plausible model was proposed

by Niels Bohr in 1913. It turns out that the **Bohr model** is wrong, but it is helpful to think about. Bohr postulated that hydrogen is made up of an electron orbiting a proton like the Earth orbits the Sun, with electrical attraction rather than gravity keeping the two particles together. He then made a (completely unjustified) guess that the angular momentum  $L$  of the electron is quantized, so that  $L = n\hbar$  for the electron, where  $n$  is an integer greater than zero and  $\hbar$  is Planck's constant  $h$  divided by  $2\pi$ . This guess implies that the electron can only have certain quantized amounts of energy, given by

$$E_n = -13.6 \frac{Z^2}{n^2} \text{ eV} \quad \text{Eq. 2}$$

where  $Z$  is the number of protons in the atom (one for hydrogen) and  $n$  is the same integer as the  $n$  in the guess that  $L = n\hbar$  for the electron. This energy is measured in "eV" or **electron-volts**, which is the energy one electron gets when accelerated through a potential of one volt (

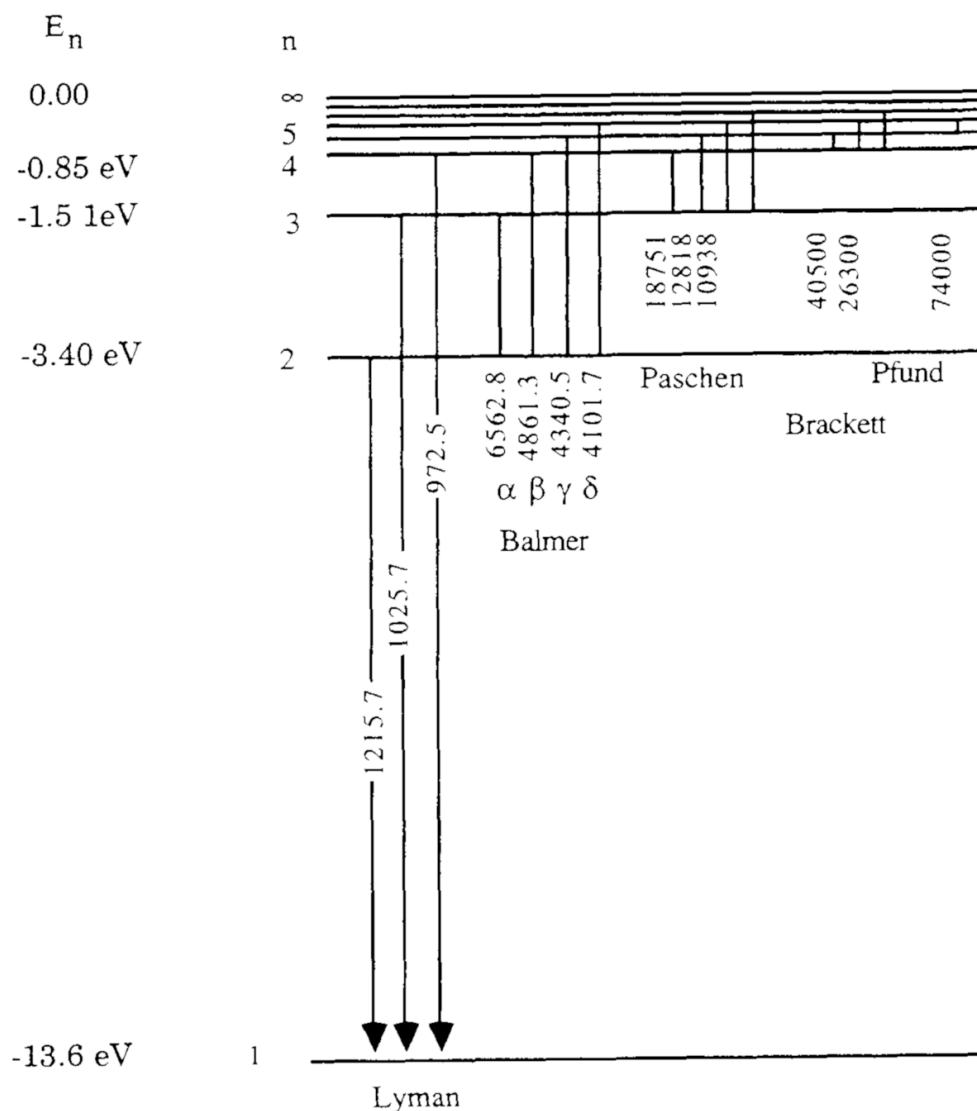
$1 \text{ eV} = 1.602 \times 10^{-19} \text{ joules}$ ).  $E_n$  is negative because the electron is bound to the proton by electrical attraction, and energy must be put into the system to tear the two particles apart and bring them to "zero" energy. Bohr included the factor  $Z$  in Eq. 2 to deal with **hydrogen-like** atoms, which have one electron "planet" orbiting  $Z$  protons stuffed into the "sun". Hydrogen-like atoms include any atom whose electrons have all but one been ionized (stripped away).

Bohr next assumed that an atom only emits light when its electron changes from one  $n$  value, or **energy level**, to another one with a lower  $n$  value and thus less energy. When this change or **transition** happens, the atom emits one photon of light whose energy is equal to the difference in energy between the two levels, satisfying conservation of energy (see Figure 2). This means that a photon emitted due to a transition from energy level  $n$  to energy level  $m < n$  will have a wavelength given by

$$\frac{1}{\lambda} = \frac{E}{hc} = \frac{E_n - E_m}{hc} = \frac{13.6 \text{ eV}}{hc} \left( \frac{1}{m^2} - \frac{1}{n^2} \right) = 1.097 \times 10^7 \text{ m}^{-1} \left( \frac{1}{m^2} - \frac{1}{n^2} \right)$$

(where  $Z = 1$  for hydrogen). This is the Balmer-Rydberg formula! The Bohr model thus says that the wavelengths of light emitted by hydrogen atoms are due to photons emitted in transitions between different energy levels  $n$  and  $m$ .





**Figure 2:** Transitions between energy levels in hydrogen with the wavelength (in angstroms,  $10^{-10} \text{ m}$ ) of the photon emitted by the transition. The transitions are grouped into "series" by the lower level they go to. Each series is named for the scientist who first observed those wavelengths in the spectrum.

While the Bohr model correctly predicts the Balmer-Rydberg formula, several of its other predictions do not agree with experiment. One of these incorrect predictions is that an atom in the lowest energy level ( $n = 1$ , also known as the **ground state**) has angular momentum  $L = \hbar$ . In fact, experiment tells us that hydrogen atoms in the ground state have  $L = 0$ . Because of this and other flaws, we have to discard the Bohr model in favor of the modern theory of quantum mechanics (QM) which agrees extremely well with all experiments performed so far. Among other things, QM correctly predicts that the angular momentum of the atom in the ground state is zero.

Students are sometimes confused about when to use the Bohr model and when to use the predictions of quantum mechanics. The answer is *always* to use QM, because it is fundamentally correct. However, in the case of hydrogen and hydrogen-like atoms, the Bohr model makes the same predictions as QM about the possible energies of the electron (Eq. 2). The Bohr model's explanation of the Balmer-Rydberg formula (Eq. 1) in terms of transitions between electron energy levels is also

essentially the same as that of QM, although QM answers the question of *why* the electron makes these transitions, which the Bohr model does not. Therefore it's O.K. to use the Bohr model when calculating the electron energy levels in hydrogen and hydrogen-like atoms, and when thinking about transitions and how they cause the hydrogen spectrum, since in these cases the Bohr model is the same as QM. In all other cases, the Bohr model is incorrect and you must use QM to get the right answer.

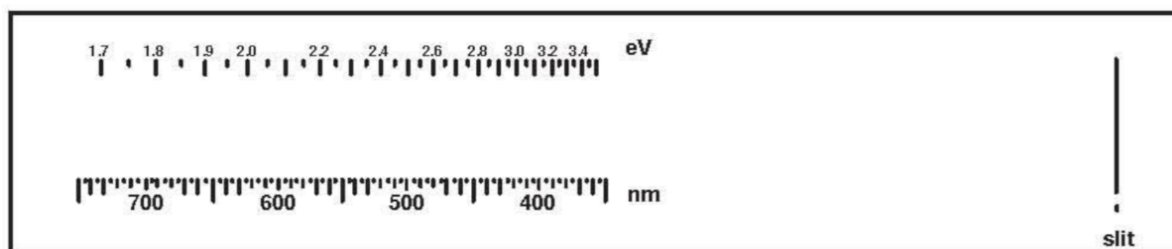
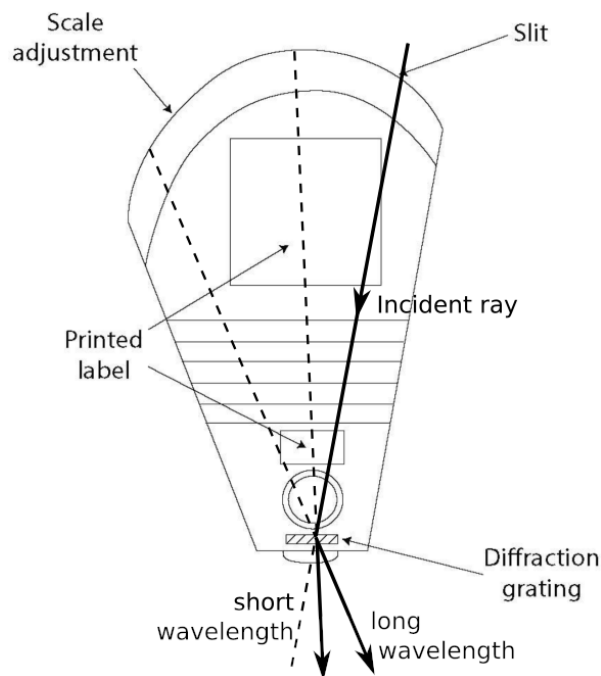
## How This Experiment Works

We're going to use gases of atoms and a **spectrometer** to look at various atomic spectra. We'll start by looking at simple spectra and measuring the Rydberg constant. Then we'll look at a more complicated spectrum that has to be dealt with quantum mechanically. Finally we'll look at the spectrum of a mixture of helium and neon gas, which will help explain how our old friend the He-Ne laser works.

## Spectrometer

The spectrometer uses a diffraction grating to separate incident light into different outgoing angles according to their wavelengths (for first-order diffraction order,  $d \sin \theta = \lambda$ , where  $d$  is separation between adjacent slits in diffraction grating,  $\lambda$  is the wavelength of light, and  $\theta$  is the outgoing angle for the light of wavelength  $\lambda$ ). The diagram on right illustrates the paths of incident light and two outgoing rays of short and long wavelengths overlaid over top view of spectrometer. As illustrated, light of different wavelengths bend at different angles going through the diffraction grating, and as you look through the diffraction grating, they appear to come from different directions, as indicated by the broken line.

The transparent scale is positioned along the wide side of the spectrometer, with roughly calibrated numbers indicating the wavelengths of light appearing to come from the direction of the scale. A diagram of the scale is shown below.



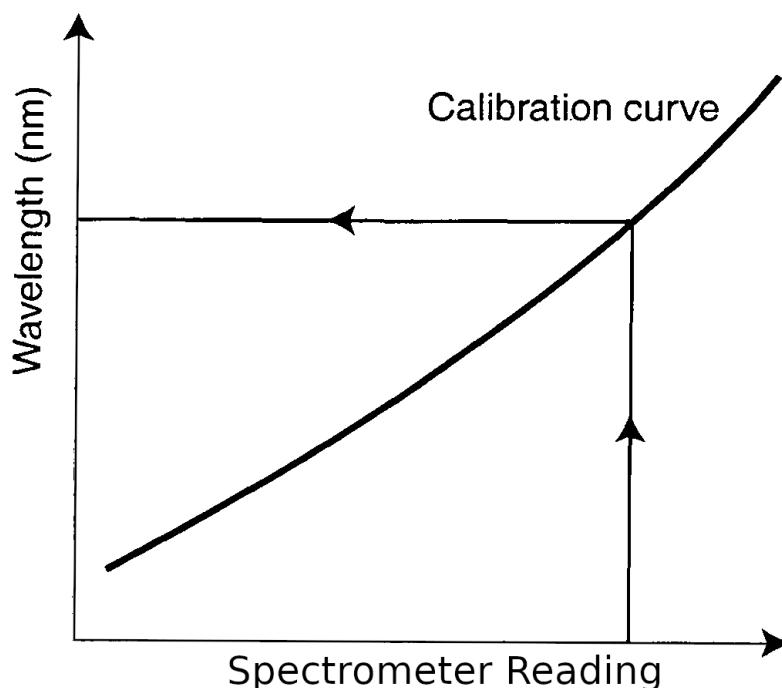
**Figure 4:** Replica of the slit and scale on the wide end of the spectrometer.

Looking through the diffraction grating, positioning the slit towards the light source so that a narrow ray of light comes through the slit incident on the diffraction grating, you can get a spectrometer reading by reading off the numbers closest to the wavelength-separated vertical columns of light.

## Calibrating the Spectrometer

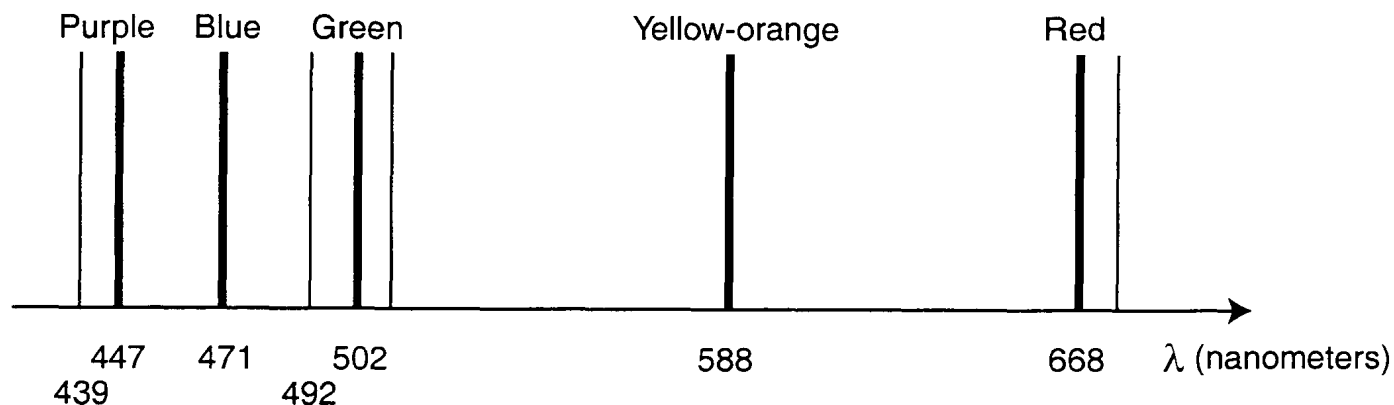
The spectrometer reading can be adjusted by sliding the scale on the wide end of the spectrometer. As you look at a reference light source through the spectrometer, you can slide the scale for a rough calibration. After your spectrometer is calibrated, please take care that the scale does not move.

A finer adjustment and calibration can be made with a use of **calibration curve**, which is a curve drawn on a plot of real wavelength versus spectrometer reading (see Figure 5). Once we have the calibration curve, we can translate a spectrometer reading into a wavelength by looking at the point on the curve corresponding to the spectrometer reading, and reading off the wavelength of that point. Note that, because the spectrometer depends on the technique of the operator, this calibration curve will be calibration curve for the spectrometer *and* the operator (consistency is key to accuracy here).



**Figure 5:** A sample calibration curve.

To make a more accurate calibration curve, we will use helium, whose spectrum contains several clearly visible wavelengths and is well-known (see Figure 6). You will take a spectrometer reading, making sure to use consistent technique, and plot the real wavelength vs. your spectrometer reading. Then, by connecting each point to the next with a straight line segment, we get a good approximation to the calibration curve for all wavelengths. The curve becomes a better and better approximation with more points, so please include as many wavelengths in the helium spectrum as you can see.



**Figure 6:** The spectrum of helium, including the wavelength and color of each bright line. Thinly-drawn lines are faint and may not be visible but are useful for identifying other lines.

The spectrometer reading (and hence the calibration curve) changes slightly depending on the measurement technique, including the angle at which the light from the atoms enters the spectrometer. Therefore, make sure to use consistent measurement technique as you are creating the calibration curve and measuring the spectrum, so that your calibration curve will remain accurate.

## Atomic Vapor Tubes

The light we will look at is from atoms sealed in glass tubes with electrodes on both ends. We have hydrogen (H), helium (He), and mercury (Hg) with the spectra available. We have other vapor tubes available, but no spectrum is provided for other vapor tubes. The atoms are in gaseous form, and by passing a current between the electrodes, we can excite them and cause them to emit light. Do not touch the thin part of the tube, which is the optically important part of the tube. As a rule, touching (including for cleaning and for necessary handling) optical elements reduce the quality of the optical element each time you touch it (so, handle it as little as possible).

## Measuring the Rydberg Constant

As shown in Figure 1, the wavelengths of the hydrogen spectrum that are in the visible range are all from transitions with  $m = 2$  (you can also see this in Figure 2 by looking at which transitions have wavelengths in the visible range). Thus if we can measure wavelength versus  $n$  for the part of the spectrum in the visible range, we can then extrapolate the value of the Rydberg constant  $R$ . There is a linear relationship between  $1/\lambda$  and  $1/n^2$  which you will determine for the Prelab.

## The He-Ne Laser Tube

The final part of the lab involves looking at the spectrum from the tube inside a He-Ne laser. You might remember warnings regarding laser radiation (on laser pointers and such); *this* light is safe to look at. What you will be seeing is a full spectrum that comes from a combination of helium spectrum and the neon spectrum (we have a neon vapor gas tube you can look at to compare). The laser tube is built to amplify one particular wavelength from that spectrum, the familiar 632.8 nm line. Most of the light that travels lengthwise along the tube and eventually exits to become the laser beam has this wavelength.

However, the light that shines off to the side of the tube, which we will look at, has many different wavelengths which the laser tube prevents from contributing to the beam.

## Procedure

### Calibration of the Spectrometer

Before we start calibrating the spectrometer, let's first look at a few light sources. First, point the spectrometer at an ambient light source. This could be a brightly illuminated wall, or light-colored object in sunlight. The slit of the spectrometer needs to be pointed at the light source. Make sure you can see the rainbow-colored spectrum overlaid over the scale.

**Q1: PREDICT:** If you look at the ceiling light in the room, what type of spectrum do you expect to see? Sketch the spectrum (does not need to be numerical) on a scale similar to that shown on Figure 4.

The spectrometer comes with some calibration instruction from the manufacturer. To roughly calibrate the spectrometer reading, follow these instructions:

- Look at a fluorescent light through the spectrometer. The spectrum from the fluorescent light should include several bright vertical "lines" (*how does this compare with your prediction?*). These are images of the slit, formed by the diffraction grating.
- The most common type of fluorescent light will have the mercury emission lines superimposed on a continuous spectrum. The green line of mercury occurs at 546 nm. If what you observe through your spectrometer does not agree with this standard value, adjust the position of the scale in your spectrometer so that it does.

Having calibrated the spectrometer roughly, now you will create a more precise calibration curve using the available helium spectrum tube.

**SAFETY NOTE:** *The spectrum tube power supply uses high voltages of about 4,000 V. Always turn off the power supply before handling the spectrum tubes.*

Place the helium tube in the light box and turn it on. Look at the helium tube through your spectrometer. Using Figure 6, locate the five bright lines of the helium spectrum and record the corresponding spectrometer readings. If you can see the other (faint) lines for which Figure 6 gives wavelengths, record their spectrometer readings as well. Plot wavelength versus spectrometer readings and connect the points by straight line segments. This is your calibration curve; from now on use it to record wavelengths (all wavelengths recorded from here on should be calibrated).

**Q2:** Now that you have your calibration curve, are the spectrometer readings on the spectrometer mostly accurate (as wavelengths in nm) or are they way off?

## The Hydrogen Spectrum

*Note: the usable life of hydrogen tube is very short; in order to keep it operating for as long as possible, do not keep the hydrogen tube on for longer than 30 seconds at a time. For extended operation, turn it off for 30 seconds for every 30 seconds it has been on.*

Turn off the light box and replace the helium tube with the hydrogen tube. Remember to use consistent technique in obtaining spectrometer readings for the hydrogen spectrum. Look at the spectrum and measure the wavelength of each visible line. There should be at least two, and probably three or more. Plot  $1/\lambda$  versus  $1/n^2$  and extrapolate the best straight line through your data points. Using this line, calculate the Rydberg constant.

**Q3:** What value do you measure for the Rydberg constant? How close is this to the given value?

## The Mercury Spectrum

Turn off the light box and replace the hydrogen tube with the mercury tube.

**Q4: PREDICT:** Will the mercury spectrum look similar to the spectrum of hydrogen? Why or why not? (Clarify for your own answer what "similar" might mean.)

Look at the mercury spectrum and compare it to the hydrogen spectrum. We have already used four wavelengths from the mercury spectrum in the Photoelectric Effect lab: 578 nm (yellow-orange), 546 nm (green), 436 nm (purple), and 405 nm (violet). Locate these lines in the spectrum and measure their wavelengths (you may not be able to see the purple or the violet line, in which case you may ignore it).

**Q5:** How accurate is your measurement of the wavelength of these lines? On the basis of this, do you think your calibration curve is a good approximation to the *true* calibration curve?

## The He-Ne Laser Spectrum

Turn off the light box and take a look at the provided He-Ne laser with exposed inner parts. Look at the laser tube **FROM THE SIDE ONLY** through the spectrometer.

**Q6: PREDICT:** Will the He-Ne spectrum have more or fewer wavelengths than the single-atom spectra of hydrogen, helium, and/or mercury? Why?

Turn on the He-Ne and look at its spectrum. Answer the questions below.

**Q7:** Is there a noticeable line measuring 633 nm in wavelength? Does the color of this line look like the color of the He-Ne laser? (Look at the beam spot from end of the laser, reflecting from a matte surface, not through the spectrometer.)

**Q8:** Place neon tube in the light box and look at the neon spectrum through your spectrometer. Compare this with what you see with He-Ne laser. Can you explain why more lines are visible for the neon spectrum, compared to the helium spectrum?

As the lab wraps up, look at other examples of spectra (some of which you might have done in Q1). Some examples of light sources to look at: (1) sunlight (if available), (2) fluorescent room light, (3) incandescent light bulb light, (4) LED light bulb light, and (5) anything else you can think of (ask me if you aren't sure if something is available).

**Q9:** How are the spectra you observe different? Based on what you know about line spectra and continuous spectra (and in-between), explain why you observe what you observe. What do you know about physical processes that produce the light you see for each of these sources?







# Lab: Cloud Chamber

**Note:** For the "lab report," answer all questions marked as **Q1**, **Q2**, etc. on **separate pieces of paper to turn in**. Use the space provided in the lab manual for graphs or answers required to be formatted in a particular way. Please follow all directions and answer all questions. You only need to turn in your answers for the lab report. Write your name and your group partners' names on your lab report.

## Introduction

Radioactivity—and the ionizing radiation produced by it—was first discovered in the late 19th century with the discovery of "Becquerel rays," with properties similar to the Roentgen ray (known today more commonly as "X-ray"). Further study over the following decade (by scientists such as Marie and Pierre Curie and Ernest Rutherford) led to categorization of ionizing radiation from radioactivity by their penetrating power: alpha, beta, and gamma rays, after the first three letters of the Greek alphabet. Further experiments demonstrated what each of these rays are:

- alpha rays (or  $\alpha$  rays): made of two protons and neutrons each (this, in fact, is the atomic nucleus of the most common helium isotope, helium-4, and the final test proving that alpha rays were helium-4 nuclei was creating helium in a chamber that has been irradiated with alpha rays); has the least penetrating power and is stopped by nearly any kind of shielding (clothes, layer of skin cells, a few centimeters of air, etc.).
- beta rays (or  $\beta$  rays): are in fact electrons, traveling at speeds anywhere from a small fraction of speed of light (similar to electrons in cathode ray tubes that create X-rays) to more than 99% of speed of light; beta rays have more penetrating power than alpha rays but less so than gamma rays; most beta rays can be stopped by a few centimeters of aluminum plate (and for many beta ray sources, the recommended shielding material is thick acrylic plate).
- gamma rays (or  $\gamma$  rays): are highly energetic photons (i.e. electromagnetic radiation), and this is the component of "Becquerel ray" that was similar to X-rays produced in cathode ray tubes; gamma rays are typically more energetic than X-rays; gamma rays have the most penetrating power and a strong gamma ray source can require inches of lead shielding, if an experimenter or a research worker needs to be exposed to it at a short distance for significant duration (the number one principle in radioactive safety is not shielding; it's **distance** and **time**—as far away as reasonably achievable and as short a duration as reasonably achievable).

One of the early tools in studying ionizing radiation (and study of elementary particles that it led to) was the [cloud chamber](https://en.wikipedia.org/wiki/Cloud_chamber)  ([https://en.wikipedia.org/wiki/Cloud\\_chamber](https://en.wikipedia.org/wiki/Cloud_chamber)). From a [review article](https://doi.org/10.1103/RevModPhys.18.225)  (<https://doi.org/10.1103/RevModPhys.18.225>) on the cloud chamber:

The Wilson cloud chamber has played a very important role in the development of modern physics. Rutherford has described the cloud chamber as "the most original and wonderful instrument in scientific history." It has also been called "the final court of appeal in physics" where many conflicting theories have been put to rest and decisions made. Where many indirect evidences fail to convince, a single cloud-chamber picture is often sufficient and carries conviction.

In this lab, you will assemble a working cloud chamber, observe its working components, test it using provided radioactive sources (these are small, weak sources that are safe to handle, if appropriate precautions are taken), and perhaps use it to observe cosmic ray particles.

## Part A: Assembling Cloud Chamber

In this part, you will assemble a working cloud chamber. A cloud chamber can be quite simple, as it is nothing more than a supercooled, saturated vapor (of something that is normally liquid at room temperature, such as water or alcohol). Imagine a morning fog (an actual inspiration for the first cloud chamber), which forms in the morning when the temperature has cooled, so there is too much water vapor in the air to remain in the gaseous form, and the extra water vapor condenses around nucleus of small particles, such as dust.

We create a similar condition except in a much smaller scale, using alcohol vapor. You will need:

- A source of alcohol vapor: the container you are provided will have absorptive material (i.e. cotton) glued to the bottom; when you pour isopropyl alcohol into the container, the soaked cotton ball will become a source of alcohol vapor.
- An airtight chamber: this is the container you are provided; once you pour alcohol into the container, you will seal it off carefully using a piece of aluminum foil and tape; it needs to be airtight so that the saturated alcohol vapor does not leak out.
- A cold reservoir: this is to cool the alcohol vapor, so that it becomes saturated (too cold for all the alcohol vapor to remain in the gas form); you will use a block of dry ice (frozen carbon dioxide).

Find the container on your table; check that it has all the required working parts; and, when you are ready, follow steps below:

### Building Saturated Alcohol Chamber

1. Pour isopropyl alcohol into the container, which has cotton glued at the bottom. Pour enough alcohol so that the cotton is visibly wet.
2. If you have any excess alcohol, pour out the excess into a provided beaker (anything that pours out is an excess; you only need alcohol that remains on the cotton).
3. Black-painted aluminum foil pieces cut into appropriate size and shape are provided. Put it over the top of the container with the black side facing inside and carefully tape around it to make an airtight seal (be careful not to tear the aluminum foil).
4. Tape over the small holes on the side of the chamber. (You may need to poke through the tape for using the needle sources. The holes are made large enough for the rod sources.)
5. Place the chamber upside down (with the aluminum foil on the bottom and alcohol-soaked cotton on top) on the cold reservoir cooled with dry ice. Let it sit for a while.

It will take a few minutes for the cold reservoir to cool the alcohol vapor in the saturated alcohol chamber. As you wait, answer the questions below.

**Q1:** Do you expect to see convection inside your chamber? Why or why not? If you do not see convection inside your chamber, will it take a relatively long time or a short period of time for the alcohol vapor to cool?

**Q2:** When the layer of oversaturated alcohol vapor forms, where do you expect to see it? Near the top, closer to the alcohol soaked cotton, or near the bottom, closer to the cold reservoir? Explain.

**Q3:** Wait and observe the cloud chamber, using the provided LED flash light. Within a few minutes, you should begin to see a fog/mist form in the chamber. Describe the fog/mist. Do you see any alcohol droplets falling from the mist?

When you see a consistent fog (and tiny alcohol droplets) forming within your chamber, you are ready to proceed to the next part. If you want, try shining a laser beam through the chamber (ask me for laser pointers). Also see where you should place the LED flash light so that various features of the mist can be best observed.

## Part B: Particle Tracks in Cloud Chamber

The clouds you see in your cloud chamber are formed when saturated alcohol vapor turns into liquid. It turns out this doesn't happen in a very clean air (with all the dust particles filtered out). In order for condensation to happen in air, it needs to be "seeded" by small dust particles—and that's the cloud and sometimes droplets of alcohol you see at the end of Part A above.

There is another way to seed cloud in your cloud chamber: if you introduce an ion (an electrically charged particle, such as a nitrogen atom that has lost an electron) into your saturated vapor, this ion will attract the alcohol molecules, and tiny alcohol droplet will form around the ion. When ionizing radiation goes through your saturated vapor chamber, the ionizing radiation will ionize air molecules along its track, and when each of these ionized air molecules seed clouds, the track of the ionizing radiation becomes visible.

We will introduce a few different types of radioactive source to your cloud chamber and observe what happens. When you are ready, obtain a strontium-90 (Sr-90) needle source from your instructor. Sr-90 is a radioactive isotope that emits beta radiation as it decays. You will first test your chamber and observe a few simple beta tracks using the Sr-90 needle source.

When you handle the needle sources, please handle it by the end opposite to the tip containing the radioactive source. Although the amount of radioactivity is small, if the radioactive source gets inside you (e.g. if the needle pokes through your skin with the radioactive end, or if you swallow the source), you may be exposed to significantly higher level of radiation than the natural background level (these exempt-quantity sources are considered safe if it *doesn't* get inside you).

Bring the Sr-90 needle source near the wall of your cloud chamber. Do you notice any changes within your cloud chamber (use LED flashlight to illuminate the chamber as you observe)? If you see the changes within your cloud chamber, confirm that this is due to the Sr-90 needle source. You can do this by moving the Sr-90 needle source away from the chamber (then your cloud chamber should return to its previous state) and back near the chamber (then you should see the change again).

If you don't see any changes, you can make them more apparent by placing the Sr-90 needle source closer to the cloud chamber: poke it through the tape covering the hole on the side and place the source (which is in the eye of the needle) directly inside the cloud chamber.

**Q4:** Describe the changes (presumably beta ray tracks) in your lab report, noting any interesting details (and whether you had to place the source inside the cloud chamber).

**Q5:** How far do beta ray tracks travel? If you didn't see anything in Q4 with the Sr-90 needle source outside the chamber (but not so far that beta rays couldn't reach it), why do you think it was? *If you can see tracks with Sr-90 source outside the chamber, try taking the needle source outside and placing it at different locations (with different material between source and chamber).*

If you observed beta ray tracks in Q4, congratulations, you have built a working cloud chamber (and your answer to Q5 may demonstrate how "shielding" ionizing radiation works).

Once you have a working cloud chamber, you can try out a couple different radioactive sources, polonium-210 (Po-210), cobalt-60 (Co-60), and cesium-137 (Cs-137). We have a limited number of these radioactive sources, so please ask me to bring it to you. I'll help you make observations with these sources for the questions below (and make sure they are available for other groups when they are ready).

If you are waiting for these radioactive sources, you may skip ahead to Part C as you wait.

**Q6:** Using the Co-60 radioactive source (we have two different types available, a disk source and a rod source; if you don't see anything with a disk source placed outside the chamber, use a rod source, which can be placed inside the chamber), make your observations of the ionizing radiation. Does it look similar to the tracks produced by Sr-90? Does it look different from the tracks produced by Sr-90? Describe it in your lab report.

**Q7:** Again, using the Co-60 radioactive source, try shielding the radiation using material and objects available in the lab. Do you still see radiation tracks in the chamber? If so, what do you think it is being produced by?

**Q8:** Using the available Cs-137 radioactive source, repeat the questions Q6 and Q7 above.

Po-210 is an alpha source that produces very little gamma ray. Using the Po-210 needle source, make your observation of the alpha ray. First, place Po-210 next to the cloud chamber wall (as we did with Sr-90, Co-60, and Cs-137). Note your observations. Then, poke the needle source through the tape over the hole; you should be able to see alpha ray tracks. Note your observations.

**Q9:** Record your observations of alpha ray tracks above in your lab notebook and explain: why do you not see alpha ray tracks going in all three-dimensional directions and instead only see tracks in a two-dimensional layer?

## Part C: Cosmic Ray Muons

There are many natural sources of radiation (collectively called "background radiation"). One particular natural source of radiation comes from the sky (or, more precisely, outer space). Although their origin is somewhat mysterious (they appear to be coming from outside the solar system, but based on calculations and observations regarding their particle energies—read about [GZK cutoff](#) <sup>↗</sup> ([https://en.wikipedia.org/wiki/Greisen%E2%80%93Zatsepin%E2%80%93Kuzmin\\_limit](https://en.wikipedia.org/wiki/Greisen%E2%80%93Zatsepin%E2%80%93Kuzmin_limit))—they can't be coming from too far away, such as outside the Milky Way galaxy), what we do know is they are mostly made up of very high energy protons bombarding Earth. Most of these high-energy protons collide with air molecules in the upper atmosphere, and out of their energy, create many exotic, unstable particles (in a manner similar to how we produce new subatomic particles in particle colliders, such as the Large Hadron Collider at CERN).

One of the most common of these particles that we can detect at sea level (i.e. where our lab is) are muons. Muons are similar to electrons (negatively charged; they are collectively called "leptons," because, of the three particle categories we have, they are the lightest, usually), except much heavier (a muon's mass is about 200 times that of an electron).

**Q10:** Let's first estimate how often we expect to see cosmic ray muons. From measurements, we know that at sea level, cosmic ray muons arrive at a rate of [1 muon per cm<sup>2</sup> per minute](#) <sup>↗</sup> ([https://cosmic.lbl.gov/SKliewer/Cosmic\\_Rays/Muons.htm](https://cosmic.lbl.gov/SKliewer/Cosmic_Rays/Muons.htm)). Estimate how many cosmic ray muons go through your cloud chamber in one minute. You will need to estimate the cross-sectional area of your cloud chamber (in the unit of cm<sup>2</sup>), and multiplying 1 muon/cm<sup>2</sup>/minute by the cross-sectional area gives how many muons go through your cloud chamber in one minute. Write your estimate in your lab report, showing your work and any assumptions made.

**Q11:** Make sure your cloud chamber is in operation (if you are in any doubt, test it with your Sr-90 source; if you can't see the beta ray tracks from your Sr-90 source, your cloud chamber is not working well); move all radioactive sources away from the cloud chamber (so that if you see any tracks made by ionizing radiation, you know it must be from cosmic ray muon), and observe your cloud chamber to see a cosmic ray muon. It may take a while, as long as several minutes (as you estimated in Q10 above, the number of muons you expect to see in one minute isn't large, and as these are random events, the actual number can be higher or lower). When you see a cosmic ray muon, describe the track made by the muon in your lab report. Does it look similar to anything else you observed in this lab so far? If it looks somewhat different, how is it different?

